Monopoly, Non-linear Pricing and Imperfect Information: The Insurance Market

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INTRODUCTION

It is well known that, when it is possible, a monopolist can increase his profits by engaging in price discrimination. For price discrimination to be feasible and desirable two conditions need to be satisfied:

(*a*) the firm must be able to identify two or more groups with different demand functions;

(*b*) it must not be possible for an " arbitrageur " to equalize the price.

Conventional theory has focused on discrimination between two localities (countries) for which the elasticity of demand differs and for which transport costs are sufficiently large to prevent price equalization.

The conventional theory of monopoly has, however, been unnecessarily restrictive in at least two important respects.

(i) First, it assumes the monopolist will charge customers an amount proportional to the quantity consumed (what I shall refer to as a linear price schedule). In fact, such a policy is almost never profit maximizing; i.e. non-linear price schedules, when feasible, are almost always desirable. For instance, if all individuals were identical the firm should charge a fixed fee—equal to the value of the consumer surplus the individual would have enjoyed had a linear price system with price equal marginal cost been employed—plus marginal cost. Not only does this increase profits but it also implies that there is no inefficiency associated with the monopoly. The monopolist introduces inefficiencies because he cannot use a non-linear price schedule, e.g. because of secondary resale markets (which are not likely to be important for many commodities) or because individuals differ and he is unable as a result to act as a perfectly discriminating monopolist. Charging any fixed fee greater than the minimum consumer surplus enjoyed by a consumer would lead to some potential customers not consuming. The firm thus has a trade-off between the number of customers and the profits per customer; and this provides the firm with an incentive to use more complicated non-linear price schedules.

(ii) Secondly, it assumes that the firm has a much more limited ability both to differentiate and discriminate among customers than is, in fact, the case. Location is not the only potential basis of differentiation. The quantity consumed of a commodity often is correlated with the consumer surplus; thus, the quantity consumed may be a basis for discriminating. Other bases may be found: the quantity of IBM cards used by a computer user may be correlated with the surplus enjoyed; the choice of an insurance contract among a set of available insurance contracts may serve to differentiate customers who differ—either in attitudes towards risk or in the risks they face. These are all examples of what I have called

elsewhere *screening devices*—a screening device is any mechanism used to differentiate among individuals—and indeed all of these are examples of the particular kind of screening device called self-selection mechanisms where the action of the individual (e.g. his choice of insurance policy) is the basis of differentiation. The trade-offs involved in the design of the screening mechanism by the monopolist are complicated and provide the basic subject of of this paper.

Most of this paper is devoted to investigating in detail the behaviour of a monopolist in an insurance market. This example is chosen because the behaviour of the competitive analogue market with imperfect information has been analysed in detail, and thus we are able to provide a detailed comparison both of the analytical structure of the problem and of the behaviour of the markets In both cases individuals are confronted with a choice of insurance contracts, and it is the choices made which provide information to the insurance company. In the competitive market there is no overall planning of the kinds of contracts offered to ensure that the information revealed (say, about the relative probabilities of an accident) are " efficiently " revealed. There was a cost in obtaining the information in that individuals with low probabilities of having an accident obtain only partial insurance. A contract was offered if, given the other contracts being offered, it could make a profit. The consequences for the profits of the other contracts were ignored.

A monopolist, on the other hand, must decide on the whole set of contracts which he makes available. He must worry not only about the information the additional contract provides and about its profitability, but also about the effect of the new contract on the profitability of old contracts; in addition, he is aware that the information provided by a new contract depends on the set of contracts already available.

As a result, as we shall see in our analyses of the insurance market, competitive and monopoly equilibrium differ in far more fundamental ways than the classical argument that the monopolist charges too high a price suggests:

(*a*) the whole set of contracts which are available will differ in the two equilibria;

(*b*) the basis for discrimination differs: in competition only differences in risk (e.g. accident probabilities) motivate the attempt to differentiate; in monopoly, anything which gives rise to a difference in demand curves—differences in risk aversion as well as differences in risk—provides a basis of differentiation.

But before turning to a detailed analysis of the insurance market we consider a more general problem of monopoly and price discrimination for which, in some special cases, results may be directly borrowed from a closely related literature: the theory of optimal taxation. (See Mirrlees [3].)

Both the problem of the monopolist and the optimal taxation problem can be viewed as screening problems. The monopolist would like to distinguish among individuals in order to charge them a different price; in the optimal tax problem the government would like to distinguish among individuals in order to tax them according to their " ability to pay ". If it had perfect information it would levy a lump sum tax on their " earning ability "; in fact, it cannot observe that directly. Hence income earned serves as a screening device. There are other possible screening devices. If there are large differences among individuals with respect to their attitudes to the leisure–consumption trade-off then the income tax may not be as good a *screening device* as perhaps a housing tax, just as in the insurance model, if attitudes towards risk differ greatly the policy purchased may not serve as a very good screening device (i.e. the accident probabilities for individuals who purchase different policies may not differ markedly). For a more extended discussion of taxation and screening, see Atkinson and Stiglitz [2].

In this paper we focus only on a limited set of potential screening devices for the monopolist, the quantity of various commodities purchased.

But even this restricted analysis provides an explanation of a number of phenomena

observed in non-competitive markets: quantity discounts, tie-in sales, commodity bundling, etc. (See also Adams and Yellen [1], and Spence [8].) The paper by Salop [5] in this symposium provides another example of an ingenious screening device.

1. A GENERAL MONOPOLY-PRICE DISCRIMINATION PROBLEM

1.1. *A General Formulation*

Consider a firm which has a monopoly over a set of commodities. There is a cost associated with producing a total quantity of the goods, $X_1, \ldots, X_m$, given by $C(X_1, \ldots, X_m)$. The jth individual has a utility function defined over his consumption of the m commodities $(x_1^j, \ldots, x_m^j) = \underline{x}^j$, and his expenditure on other goods, which we shall denote by x_0^j. Different individuals differ in a systematic and continuous way (say in income); thus, we write the utility of an individual of type θ as:

$$U^j = U^j(x_0^j, \ldots, x_m^j;\ \theta).$$

The problem of the monopolist is to find a *revenue function* $R(x_1, \ldots, x_m)$ giving the payment which a consumer who purchases a vector of goods $x_1, \ldots, x_m$ from the firm must pay the firm, which maximizes firm profits. For any particular revenue function the jth consumer maximizes his utility.

$$\max U^j(x_0^j, x_1^j, \ldots, x_m^j;\ \theta) \qquad \ldots(1.1)$$

subject to his budget constraint

$$R(\underline{x}^j) + x_0^j = W_0^j(\theta), \qquad \ldots(1.2)$$

where W_0^j represents total available wealth.

The solution to this yields a vector of consumption for the θ type individual $\underline{x}^j(\theta;\ R)$, which is written to remind us of the dependence of the consumption vector on the payments function R. Total profits of the firm are then

$$\pi = \int R(\underline{x}^j(\theta;\ R)dF(\theta)) - C\left(\int \underline{x}^j(\theta;\ R)dF(\theta)\right) \qquad \ldots(1.3)$$

where $F(\theta)$ is the distribution of individuals according to the parameter θ.

1.2. *Special Cases*

This general formulation includes several special cases which have been dealt with elsewhere in the literature.

(*a*) Conventional monopoly theory has focused on the case where R is separable and linear, i.e.

$$R(\underline{x}^j) = \sum_i p_i x_i^j$$

(*b*) Commodity discounts would be consistent with

$$R(\underline{x}^j) = \sum_i r_i(x_i^j)$$

with $r_i'' < 0$.

(*c*) Tie-in sales and bundling are associated with non-separable payment functions. If the number of units of a tied bundle which are consumed by a particular individual is not observable

$$R(\underline{x}^j) = x_1^j r\left(\frac{x_2^j}{x_1^j}, \frac{x_3^j}{x_1^j}, \ldots, \frac{x_m^j}{x_1^j}\right).$$

Consider the case of two commodities. If separate purchases are not observable, i.e. there is no way that a monopolist can prevent a customer from mixing two bundles, then the

function R is restricted to functions which are quasi-convex, as depicted in Figure 1 (a). If tie-in sales or bundling is not desirable then $R(x_1, x_2) = \bar{R}$ defines a straight line as depicted in Figure 1 (b). Strict bundling—the availability of only a package $(\hat{x}_1, \hat{x}_2)$ is illustrated in Figure 1 (c).

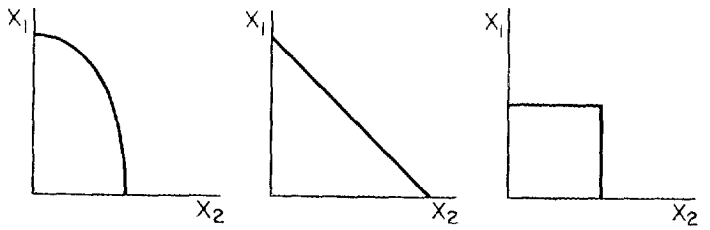

FIGURE 1 (a) Bundles purchasable for a dollar.

FIGURE 1 (b) No tie-in sales or bundling.

FIGURE 1 (c) Strict bundling.

1.3. *Analogy to Optimal Tax Problem*

It should be obvious that the problem of the monopolist is exactly analogous to the general optimal taxation problem formulated by Atkinson and Stiglitz [2]. There, the government had to find a tax function $T(x_1, \ldots, x_m) = T(\underline{x})$ which maximized social welfare subject to the government's attaining the desired level of revenues. For simplicity, let the competitive prices of all commodities be constant and normalized at unity. Then the consumer maximizes his utility subject to the budget constraint

$$T(\underline{x}^j) + \sum_{i=0}^{m} x_i^j = W_0^j,$$

yielding a vector of consumption of $\underline{x}^j(\theta;\ T)$. Total revenues of the government are then $T(\underline{x}^j(\theta;\ T))$. Consider the problem of maximizing a weighted average of consumers' welfare (say, as represented by a utilitarian social welfare function) and government revenues,

$$\max \lambda \int T(x^j(\theta))dF + (1-\lambda) \int U^j(x^j(\theta);\ \theta)dF.$$

It is immediate that the monopolist's problem is the special case of this with $\lambda = 1$, whereas in the government's problem the value of λ depends on the magnitude of the revenue which the government must raise.

1.4. *Some Properties of the Optimal Payments Function*

Making use of (1.2) and our assumption of constant unit costs (normalized at unity) we rewrite (1.3) to read:

$$\max \int \left\{ W_0^j(\theta) - \sum_{i=0}^{m} x_i^j(\theta) \right\} dF(\theta). \qquad \ldots(1.4)$$

In addition we assume that U is thrice differentiable and there are no inferior goods. Let U^j be our state variable and $x_1^j, \ldots, x_m^j$ our control variables;

$$\frac{dU^j}{d\theta} = \frac{\partial U^j}{\partial \theta} + U_0^j \frac{\partial W_0^j}{\partial \theta}. \qquad \ldots(1.5)$$

Then our Hamiltonian can be written as

$$H = \left\{ W_0^j(\theta) - \sum_{i=0}^{m} x_i^j(\theta) \right\} f(\theta) + \mu \left[\frac{\partial U^j}{\partial \theta} + U_0^j \frac{\partial W^j}{\partial \theta} \right], \qquad \ldots(1.6)$$

where $f(\theta)$ is the density function of θ. Then,

$$\frac{\partial H}{\partial x_i^j} = f(\theta)\left(\frac{U_i^j}{U_0^j} - 1\right) + \mu\left\{\frac{\partial^2 U^j}{\partial\theta\partial x_i^j} + U_{0i}^j\frac{\partial W^j}{\partial\theta} - \frac{U_i^j}{U_0^j}\left[\frac{\partial^2 U^j}{\partial\theta\partial x_0^j} + U_{00}^j\frac{\partial W^j}{\partial\theta}\right]\right\} = 0. \qquad \ldots(1.7)$$

The first-order conditions for utility maximization imply that

$$R_i \equiv \frac{\partial R}{\partial x_i} = \frac{U_i^j}{U_0^j}. \qquad \ldots(1.8)$$

Substituting (1.8) into (1.7) and using the differential equation for $\mu(\theta)$:

$$-\frac{d\mu(\theta)}{d\theta} = -\frac{f(\theta)}{U_0^j} + \frac{\mu}{U_0^j}\left(\frac{\partial^2 U^j}{\partial\theta\partial x_0^j} + U_{00}^j\frac{\partial W^j}{\partial\theta}\right) \qquad \ldots(1.9)$$

and the boundary value conditions we can solve in principle for the optimal payments function.

Here we focus on some special cases:

Individuals differ only in wealth. Without loss of generality we let $\partial W^j/\partial\theta = 1$. Then

$$-\left(\frac{R_i - 1}{R_i}\right) = -\frac{\mu}{f}\left(U_{00}^j - \frac{U_0^j}{U_i^j}U_{0i}^j\right). \qquad \ldots(1.10)$$

Note that

$$\frac{(R_i-1)/R_i}{(R_k-1)/R_k} = \frac{U_{00}^j - \dfrac{U_{0i}^j U_0^j}{U_i^j}}{U_{00}^j - \dfrac{U_0^j U_0^j}{U^j}}, \qquad \ldots(1.11)$$

the relative mark-up over marginal costs between any two commodities sold by the monopolist, depends only on properties of the utility function, not on the density distribution of the population (f).

Letting $\mathrm{MRS}_{0i} = U_0/U_i$, the marginal rate of substitution between good 0 and good i, (1.11) can be re-written as (where, on taking the partial derivatives, it is understood that $x_1, \ldots, x_m$ are kept constant)

$$\frac{R_i - 1}{R_k - 1} = \frac{\partial \mathrm{MRS}_{0i}/\partial W_0}{\partial \mathrm{MRS}_{0k}/\partial W_0} \qquad \ldots(1.11')$$

the relative mark-up depends on how the marginal rate of substitution is affected (relatively) by a change in wealth (or x_0).

Next consider the still further specialization of constant marginal utility of income. Then it is optimal to charge price equal to marginal cost; all of the monopoly power is exercised by having a fixed fee to purchase from the monopolist. (This is not surprising, since then all individuals who purchase from the firm purchase the same amount.)

More generally we have under fairly weak conditions that the poorest and richest individuals are charged, at the margin, marginal cost: $R_i(\underline{x}(\underline{\theta})) = R_i(\underline{x}(\infty) = 1$, when when $\underline{x}(\underline{\theta})$ and $\underline{x}(\infty)$ stand for the consumption vector of the poorest individual who purchases the commodity and the richest individual, respectively.[1] Thus, for small purchases there must be quantity premia; for large purchases quantity discounts. If repeated purchases are unobservable, there cannot be quantity premia and hence the price schedule will entail a fixed charge, a constant price up to a point and quantity discounts beyond that.

Individuals differ in marginal utilities of income. For simplicity we assume the monopolist produces a single good, x_1. We assume the utility function takes on the special form

$$U = z(x_1) + \theta x_0 = z(x_1) + \theta(W_0 - R(x_1))$$

and W_0 is the same for everyone, but θ, the marginal utility of income, differs. For each individual, however, θ is a constant. Then (1.7) to (1.9) become:

$$z' = \theta R' \qquad \ldots(1.8')$$

$$\frac{R'-1}{R'} = \frac{\mu}{f} \qquad \ldots(1.7')$$

$$-\frac{d\mu}{d\theta} = \frac{-f+\mu}{\theta}. \qquad \ldots(1.9')$$

Then the firm should charge a fixed fee and a constant price per unit if, and only if, μ/f is a constant, i.e.

$$\frac{1}{\mu}\frac{d\mu}{d\theta} = \frac{f}{\mu\theta} - \frac{1}{\theta} = \frac{f'}{f}$$

i.e.

$$\frac{\theta f'}{f} = \left(\frac{f}{\mu} - 1\right)$$

or

$$f = a\theta^{\gamma}.$$

We assume

$$0 \leqq \theta \leqq \left(\frac{1+\gamma}{a}\right)^{1-\gamma}.$$

Hence

$$\frac{R'-1}{R'} = \frac{1}{1+\gamma}$$

The fixed entry fee, c, is chosen to maximize profits. For the marginal entrant, $\bar{\theta}$,

$$U(x^*(\bar{\theta})) + \bar{\theta}(W_0 - c - R'x^*) = z(0) + \bar{\theta}W_0.$$

Profits are:

$$\int_0^{\bar{\theta}} \{(R'-1)x^* + c\} f(\theta) d\theta$$

so profits are maximized when

$$1 - \frac{x^*(\bar{\theta}))}{R(x^*(\bar{\theta}))} = \frac{1}{1+\gamma}.$$

Individuals differ in tastes. We further specialize the model by assuming that (a) purchases and sales cannot be monitored, and (b) the monopolist controls two commodities, x_1 and x_2, which are required in fixed proportions by each individual, but the individuals differ in the proportions required. Assumption (a) implied that the revenue function must be of the form

$$R(x_1, x_2) = x_1 R(1, x_2/x_1) + c(x) = x_1 r(x) + c(x),$$

where $x \equiv x_2/x_1$ and $c(x)$ is the fixed charge for buying bundle x.

We shall further simplify by assuming that the fixed charge is arbitrarily set to zero ($c(x) \equiv 0$). This considerably simplifies the analysis and yields results that are more easily interpretable. With that restriction, $r'' \geqq 0$, for if $r''<0$ over some interval $\underline{x} \leqq x \leqq \bar{x}$, then any bundle in the interval can be purchased more cheaply by buying, say, bundles $\underline{x}$ and $\bar{x}$, and mixing. The second assumption implies that we can write the θ individual's utility as

$$U = U(\min(\theta x_1, x_2), W - R(x_1, x_2), \theta) = U(x_1(\min(\theta, x), W - x_1 r(x), \theta).$$

The first-order conditions give $x = \theta$ and $U_0/U_1 = r(x)/\theta$, from which we can calculate the demand curve

$$x_1(\theta) = D(r(\theta), \theta).$$

The profits of the firm are then (under the earlier normalization that it costs one unit of our numeraire to produce one unit of x_1 and one to produce one unit of x_2)

$$\int_0^\infty D(r(\theta), \theta)(r(\theta)-(1+\theta))dF(\theta)$$

subject to the constraint that

$$r' \geqq 0, \quad r'' \geqq 0.$$

Ignoring the constraint we obtain

$$\frac{r-(1+\theta)}{r} = \frac{1}{\eta(\theta)} \quad \text{or} \quad r = \frac{\eta(1+\theta)}{\eta-1},$$

where $\eta(\theta) = -D_r r/D$, the elasticity of demand; the mark-up over marginal cost $(1+\theta)$ is inversely proportional to the demand elasticity.

In Figure 2 we have plotted $\eta(\theta)(1+\theta)/(\eta(\theta)-1)$.

In Figure 2 (a), it is an increasing convex function, so the curve plotted satisfies the constraint and gives the optimal $r(x)$ function. In Figure 2 (b), there is an interval over which the function is concave; the optimal $r(x)$ function then consists of a strictly convex portion joined to a linear portion as in the figure.

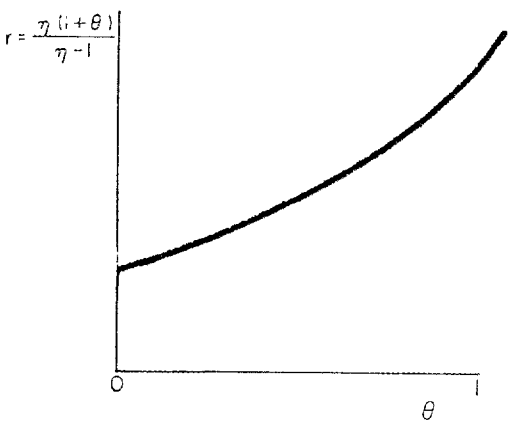

FIGURE 2 (a)

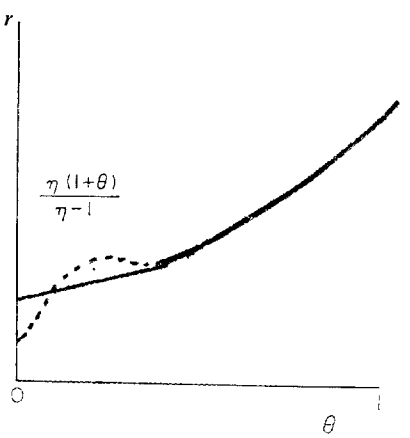

FIGURE 2 (b)

2. THE INSURANCE MARKET

The remainder of this paper is devoted to examining monopoly with imperfect information in a specific market: the market for accident insurance. This market is chosen for two reasons: (i) since the corresponding competitive market has been analysed, we can make precise comparisons between monopoly and competition with imperfect information; (ii) there are, for this market, natural parameterizations describing how individuals differ from one another.

The basic model is the same as that presented for the competitive economy in Rothschild–Stiglitz [4], where there is a more detailed discussion of the assumptions of the model. All individuals are assumed to be identical, except with respect to the probability of having an accident. They have an initial endowment of W_0; the loss, if an accident occurs, is d, and the probability of an accident is p. Their expected utility in the absence of insurance is thus

$$\mathscr{W} = U(W_0-d)p+U(W_0)(1-p). \qquad \ldots(2.1)$$

An insurance policy pays an amount, β, in the event of an accident; in turn, individuals must pay the insurance firm an amount α, if there is no accident. (This does not quite correspond to the conventional characterization of an insurance policy, which entails a premium paid in all states of nature and a receipt from the firm only in the state of nature " accident ", i.e. in the conventional terminology, α would be the premium and $\alpha+\beta$ the benefit.) Thus the individual's expected utility with an insurance policy (α, β) is

$$U(W_0-d+\beta)p+U(W_0-\alpha)(1-p) \equiv V(\alpha, \beta;\ p). \qquad \ldots(2.2)$$

The individual chooses from the available set of insurance policies that policy which maximizes his expected utility V. Let $\{\alpha(p), \beta(p)\}$ denote the policy chosen by an individual of type p. The insurance company's expected profit on that policy is clearly

$$\pi(p) = (1-p)\alpha(p)-\beta(p)p. \qquad \ldots(2.3)$$

Let $F(p)$ be the distribution function of individuals in the population. For simplicity, let us normalize the size of the population at unity. Then the expected profits of the insurance company are just

$$\bar{\pi} = \int \pi(p)dF(p). \qquad \ldots(2.4)$$

We assume that the insurance company is risk neutral and hence it wishes to choose a set of contracts to maximize expected profits. We are concerned here with the characterization of the solution to this problem.

In the next section we analyse the solution for the two-group case; in the following section we consider the case where there is a continuum of groups.

3. TWO GROUPS DIFFERING ONLY IN PROBABILITY OF ACCIDENT

The advantage of considering the case where there are only two groups in the population, differing only in the probability of having an accident, is that we can neatly characterize the solution diagrammatically. For each individual, there are only two states of nature, " Accident " and "No Accident ". Wealth in the former state is W_0-d, in the latter, W_0. The point E in Figure 3 (a) (and subsequent diagrams) represents this initial " endowment ". As we noted earlier, an insurance contract may then be viewed as a promise by the individual to pay an amount α to the firm if he has no accident, in return for a promise by the insurance to pay β if he does have an accident. Thus, with insurance contract $\{\alpha, \beta\}$ the individual's wealth in the state of nature " no accident " is $W_0-\alpha$, and in the state of nature " accident " is $W_0-d+\beta$. Thus, any point to the north-west of E (below the 45° line) represents the wealth of the individual after having purchased some insurance contract with $\alpha>0$, $\beta>0$.[2]

For instance, the point C_H corresponds to a contract with benefit α_H and premium β_H. For simplicity, we shall sometimes refer to the point C_H as " the contract C_H " rather than as the wealth generated by the contract $\{\alpha_H, \beta_H\}$.

Variables pertaining to the high-risk individuals will be denoted by the subscript H, for the low-risk individuals by the subscript L.

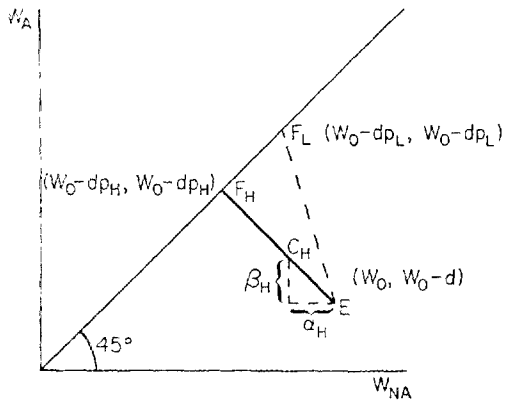

FIGURE 3 (a)

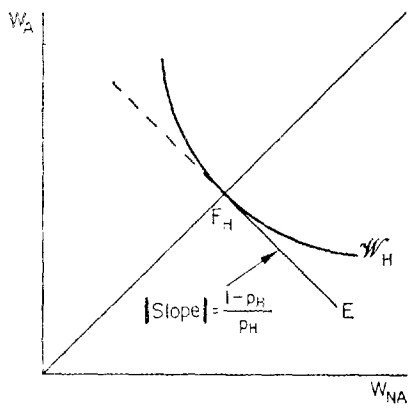

FIGURE 3 (b)
Basic diagrams.

The set of contracts which just breaks even (on average) for the high-risk individuals is given by

$$\alpha_H(1-p_H)-\beta_H p_H = 0$$

and is depicted in Figure 3 (a) by the line EF_H. The point F_H represents the expected wealth of the high-risk individual, i.e. $W_0-p_H d$. Similarly, the set of contracts with a given level of expected profits, $\bar{\pi}_H$, is given by

$$\alpha_H(1-p_H)-\beta_H\beta_H-\bar{\pi}_H = 0,$$

i.e. lines parallel to EF_H. Clearly, lines above EF_H represent higher benefits for the same premium, so expected profits are negative, and conversely for lines below EF_H.

The corresponding line for the low-risk individual is denoted by EF_L. F_L clearly lies above F_H, since the expected wealth of the low-risk individual, $W_0-p_L d$, is greater than that of the high-risk individual.

Finally, in Figure 3 (b) we have drawn the indifference curves of the individuals. There are two critical aspects of these indifference curves. First, observe that the marginal rate of substitution (the slope of the indifference curve) along the 45° line is just equal to the ratio of the probability of not having an accident to that of having an accident,

$$-\left.\frac{\partial W_A}{\partial W_{NA}}\right|_{\mathscr{W}} = \frac{U'(W_{NA})(1-p)}{U'(W_A)p} = \frac{(1-p)}{p}$$

when $W_{NA} = W_A$ (where W_A is wealth in state of nature " accident ", W_{NA} is wealth in state of nature " no accident "), which in turn is the same as the slope of the constant profits curve for the group of individuals with those accident probabilities. Secondly note that so long as everyone has the same degree of risk aversion, the absolute value of the slope of the high-risk individual's indifference curve through any point is less than that of the low-risk individual's indifference curve through the same point:

$$\frac{U'(W_{NA})(1-p_H)}{U'(W_A)p_H} < \frac{U'(W_{NA})(1-p_L)}{U'(W_A)p_L}.$$

For purposes of comparison, it may be useful to recall the equilibrium for (*a*) competition with perfect information about accident probabilities, (*b*) competition with imperfect information and (*c*) monopoly with perfect information.

Competition with perfect information leads to each individual paying his own " actuarial odds " (since we have assumed that the insurance companies are risk neutral) and obtaining complete insurance. Indeed, for this solution to emerge, we do not require that the individual know his own probabilities of having an accident, only that the insurance firm can, somehow, ascertain those probabilities. Thus, in Figure 4 (a) the points C_H and C_L denote the equilibrium contracts; the point on the zero profit line for the respective groups which maximizes expected utility occurs along the 45° line; at that point the indifference curve and the zero profit line have the same slope.

In competitive markets with imperfect information, if an equilibrium existed, it always entailed there being a separate contract for each of the two groups; there was never any " pooling ". The high-risk group obtained complete insurance, the low-risk group partial insurance. But both groups purchased some insurance. Figure 4 (b) depicts the equilibrium for this case. The high-risk individual is at exactly the same point he was with perfect information. The low-risk individual, however, purchases the " best " contract which breaks even and which will not, at the same time, be purchased by the high-risk individual, given that he has the option of buying the contract C_H. The low-risk individuals avoid

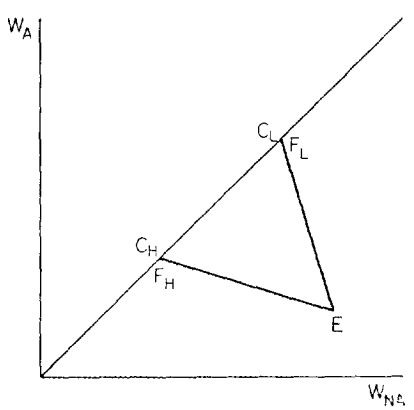

FIGURE 4 (a)
Competitive equilibrium: perfect information.

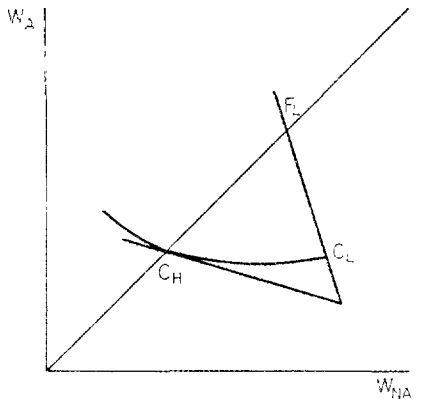

FIGURE 4 (b)
Competitive equilibrium: imperfect information.

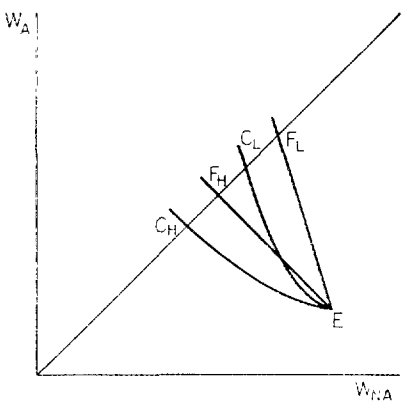

FIGURE 4 (c)
Monopoly: perfect information.

" subsidizing " the high-risk individuals (as they would if they purchased the same insurance contract) but only at the " price " of purchasing only partial insurance. Since the high-risk individuals *know* that they are high risk, it is far more important for them to obtain " complete coverage " than it is for the low-risk individuals. This enables a " separation " of the market according to risk groups.

Finally, in monopolistic markets with perfect information, the monopolist offers the contract to each individual which maximizes his expected profit. This is the contract with complete insurance at terms which leave the individual " just indifferent " between purchasing the policy and having no insurance, i.e. the monopolist is able to extract all the consumer surplus involved in the reduction of risk. (This is easy to see, since the set of contracts which the individual will purchase (rather than have no insurance) is given by the points which lie above the indifference curve through E. The iso-expected profits lines are parallel, say, for the high-risk individuals, to EF_H. Again, the indifference curve is tangent to an iso-expected profit line at the 45° line.) Figure 4 (c) depicts the solution with the two contracts C_H and C_L.

The characterization of the monopolistic equilibrium with imperfect information is somewhat more difficult. We shall show that, as in the other three cases analysed, the same contract will never be purchased by both high- and low-risk individuals, i.e. again there

never exists a pooling equilibrium. But now the low-risk individual may not purchase any insurance at all; the high-risk individual, however, always purchases complete insurance. This analysis provides an explanation for the fact that low-risk individuals often would like to buy partial insurance at " favourable terms " but are unable to do so; the reason is that at any set of terms which they would find favourable, the high-risk individuals will switch to the partial insurance and the profits of the firm will be lowered.

More formally, we establish the following properties of the equilibrium:

Property 1. *The optimal contract for the high-risk individual is complete insurance, if the high- and low-risk individuals buy separate insurance.*

Proof. The solution may be seen very easily diagrammatically in Figure 5. We are constrained to choosing contracts which are preferred to the contract purchased by the low-risk group (C_L) by the high-risk individuals, but which are not preferred by the low-risk individuals. The set of such points is shaded in Figure 5 (a). Since the slope of the indifference curve is equal to $(1-p)/p$, the slope of the " constant profits " line, at the 45° line, but elsewhere it is flatter, clearly profits are maximized by the contract which is at the intersection of the 45° line and the high-risk individuals' indifference curve through C_L.

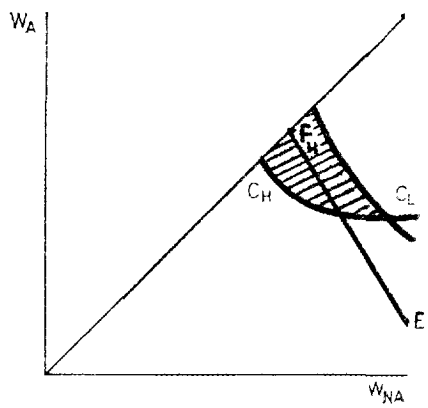

FIGURE 5 (a)

For separation, we require

$$U(W_0-d+\beta_H)p_H+U(W_0-\alpha_H)(1-p_H) \geqq U(W_0-d+\beta_L)p_H+U(W_0-\alpha_L)(1-p_H). \quad \text{...(3.1a)}$$

and

$$U(W_0-d+\beta_L)p_L+U(W_0-\alpha_L)(1-p_L) \geqq U(W_0-d+\beta_H)p_L+U(W_0-\alpha_H)(1-p_L) \quad \text{...(3.1b)}$$

(3.1) says that each type of individual prefers his contract to the other contract. Clearly, profit maximization on the part of the monopolist must entail (3.1) holding with equality. Otherwise, there exists a contract $\{\alpha'_H, \beta'_H\}$ such that

$$\alpha'_H(1-p_H)-\beta'_H p_H > \alpha_H(1-p_H)-\beta_H p_H$$

and (recalling the definition of V from (2.2))

$$V(\alpha_L, \beta_L, p_H) \leqq V(\alpha'_H, \beta'_H, p_H) \leqq V(\alpha_H, \beta_H, p_H) \leqq V(\alpha_H, \beta_H, p_L) \leqq V(\alpha_L, \beta_L, p_L).$$

The choice of contract of the low-risk individual will be unaffected as we change contracts, so long as (3.1) holds; and so long as it holds with inequality, we can increase the premium (α_H), without changing the benefits, and hence increase the profits. There is one minor

technicality: how do we know as we increase the premiums that the high-risk group will continue to buy insurance? Because it is easy to establish that if

$$V(0, 0, p_L) < V(\alpha_L, \beta_L, p_L)$$

then

$$V(0, 0, p_H) < V(\alpha_L, \beta_L, p_H) (\leqq V(\alpha_H, \beta_H, p_H)).$$

Profits may be written

$$\bar{\pi} = [\alpha_H(1-p_H) - \beta_H p_H]N_H + [\alpha_L(1-p_L) - \beta_L p_L]N_L, \qquad \ldots(3.2)$$

where N_L is the number of low-risk individuals, N_H the number of high-risk individuals.

Hence, if we maximize profits subject to (3.1) we obtain

$$\frac{d\bar{\pi}}{d\beta_H} = \left[(1-p_H)\frac{U'(W_0 - d + \beta_H)p_H}{U'(W_0 - \alpha_H)(1-p_H)} - p_H\right]N_H = 0, \quad \text{when} \quad \alpha_H = d - \beta_H. \qquad \ldots(3.3)$$

Thus, we must have complete insurance for the high-risk individual (just as we did in the competitive market).

Property 2. *If the low-risk individual buys insurance which is different from that of the high-risk individual his level of utility is " essentially " the same as it would have been had he not purchased any insurance.*

Proof. We assume the contract for the high-risk individual, C_H, is given. Following Property 1, it must lie along the 45° line. Thus, the set of contracts which the monopolist can choose and which " separate " the groups are the points below the high-risk individual's

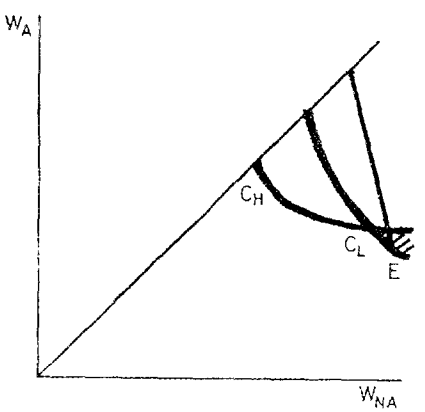

FIGURE 5 (b)

indifference curve through C_H and above the low-risk individual's indifference curve through E (the no insurance point). Again using the fact that the indifference curves are flatter than the line of constant profits, profits are maximized at C_L, the intersection of the two indifference curves (fig. 5 (b)).

Property 3. *High-risk and low-risk individuals never purchase the same policy.*

Proof. To show this, we show that there exists a pair of policies one of which will be purchased by each group, which will increase profits.

In Figure 6, we have labelled a " pooling contract ", i.e. one which is purchased by both groups, by C_P. Assume C_P does not entail complete insurance. There are two possibilities. At C_P, the high-risk individuals are in effect being subsidized, or the firm is

making a profit even on the high-risk individuals. In the latter case, consider the pair of contracts, such that C_H involves complete insurance and the high-risk individual is indifferent between C_H and C_P, and $C_L = C_P$. Using the fact noted above that the slope of the low-risk individual's indifference curve through any point is greater than that of the high-risk individual's, it is clear that profits on the high-risk individual are increased, and on the low-risk individuals unchanged.

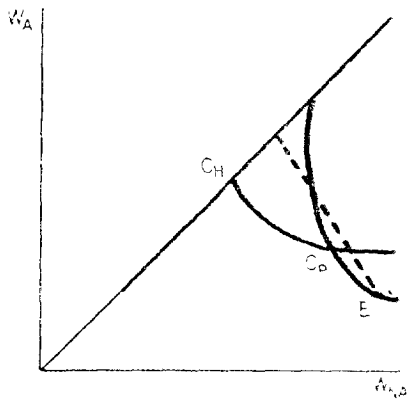

FIGURE 6 (a)

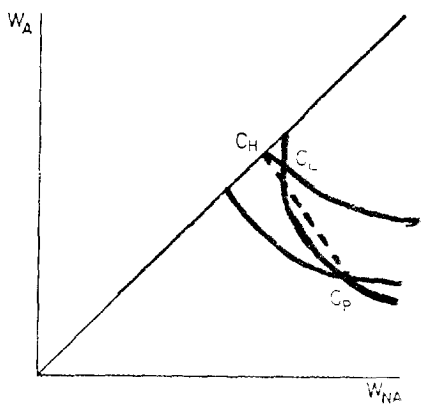

FIGURE 6 (b)

In the former case, we show that the implicit subsidy from the low-risk to the high-risk individuals is " inefficient ". The value of the subsidy (the negative profit on the contracts purchased by high-risk individuals) is constant along a line through C_P which has a slope of $1-p_H/p_H$ (fig. 6 (b)). Thus, let the firm offer the contract C_H, involving complete insurance, with an equal effective subsidy, and the contract C_L, the intersection of low risk individual's indifference curve through C_P and the high risk individual's indifference curve through C_H (see Property 2). Clearly, profits on C_L exceed those on C_P purchased by low-risk individuals.

This establishes that if there is a pooling contract, it must provide complete insurance. But direct calculation establishes that the best pooling contract cannot involve complete insurance: clearly, if there is a single policy purchased by both groups

$$V(0, 0, p_H) \leqq V(\alpha, \beta, p_H) \qquad \text{...(3.4a)}$$

$$V(0, 0, p_L) \leqq V(\alpha, \beta, p_L). \qquad \text{...(3.4b)}$$

If (3.4b) is satisfied (3.4a) is. Moreover, profit maximization entails (3.4b) holding with equality. (Otherwise we could increase premiums without changing benefits and the policy would still be purchased.) Thus we must maximize

$$\bar{\pi} = \alpha(1-\bar{p}) - \beta\bar{p} \qquad \ldots(3.5)$$

subject to

$$U(W_0-d)p_L + U(W_0)(1-p_L) = U(W_0-d+\beta)p_L + U(W_0-\alpha)(1-p_L),$$

i.e.

$$\frac{d\bar{\pi}}{d\beta} = \frac{(1-\bar{p})p_L U'(W_0-d+\beta)}{(1-p_L)U'(W_0-\alpha)} - \bar{p} = 0, \qquad \ldots(3.6)$$

whence

$$\frac{U'(W_0-d+\beta)}{U'(W_0-\alpha)} = \frac{\bar{p}/(1-\bar{p})}{p_L/(1-p_L)} > 1.$$

There remains then only one problem for the monopolist: what policy to offer the high-risk individual, i.e. he will offer complete insurance, but he may offer any contract between that which just induces the high-risk individual to purchase insurance (in which case the low-risk individual clearly will purchase no insurance) and that where the low-risk individual is just indifferent to buying no insurance and the given policy. Obviously, if almost all the individuals in the population are high risk, then the first gives the maximum profit; if all the individuals in the population are low risk, the second gives the maximum profit. What is more interesting, however, is

Property 4. *There exists a critical (finite) ratio of high- to low-risk individuals, such that if the actual ratio exceeds the critical ratio, low-risk individuals purchase no insurance.*

Proof. (α_L, β_L) is given by the solution to the set of equations

$$\begin{aligned} U(W_0-\alpha_H) &= U(W_0-d+\beta_L)p_H + U(W_0-\alpha_L)(1-p_H) \\ U(W_0-d+\beta_L)p_L + U(W_0-\alpha_L)(1-p_L) &= U(W_0-d)p_L + U(W_0)(1-p_L), \end{aligned} \qquad \ldots(3.7)$$

where

$$\alpha_H = d - \beta_H.$$

Thus total profits are

$$\bar{\pi} = N_H\{\alpha_H(1-p_H) - p_H\beta_H\} + N_L\{\alpha_L(1-p_L) - p_L\beta_L\} \qquad \ldots(3.8)$$

so

$$\begin{aligned} \frac{d\bar{\pi}}{d\beta_H} &= -N_H + N_L\left\{\frac{d\alpha_L}{d\beta_L}(1-p_L) - p_L\right\}\frac{d\beta_L}{d\beta_H} \\ &= -N_H + N_L\left\{\left(\frac{U'(W_0-d+\beta_L)}{U'(W_0-\alpha_L)} - 1\right)\frac{U'(W_0-d+\beta_H)}{U'(W_0-d+\beta_L)}\,\frac{p_L(1-p_L)}{p_H(1-p_L)-(1-p_H)p_L}\right\}. \end{aligned} \qquad \ldots(3.9)$$

Thus if

$$\left(\frac{d\bar{\pi}}{d\beta_H}\right)_{\beta_L=\alpha_L=0} < 0, \qquad \ldots(3.10)$$

then the low-risk individuals purchase no insurance, i.e. if

$$\frac{N_H}{N_L} > \left(\frac{U'(W_0-d)}{U'(W_0)} - 1\right)\frac{U'(W_0-d+\beta_H)}{U'(W_0-d)}\,\frac{p_L(1-p_L)}{p_H(1-p_L)-(1-p_H)p_L} \qquad \ldots(3.11)$$

the low-risk individuals purchase no insurance. The magnitude of the critical ratio depends on the magnitude of the differences in their accident probabilities, the size of the accident, and the degree of risk aversion. For a logarithmic utility function we obtain

$$\frac{N_H}{N_L} > \frac{d}{(W_0-d)^{p_H}W_0^{(1-p_H)}}\,\frac{1}{\dfrac{p_H}{p_L} - \dfrac{(1-p_H)}{(1-p_L)}}. \qquad \ldots(3.12)$$

If $p_H = 0{\cdot}5$, $p_L = 0{\cdot}25$, $d/W_0 = 0{\cdot}25$, the critical ratio is $\sqrt{3}/8$, i.e. if more than 1/4 of the population belongs to the high-risk group, no insurance will be purchased by the low-risk group.

TABLE I

Comparison of equilibria

	Competition		Monopoly	
	Perfect information	Imperfect information	Perfect information	Imperfect information
High risk	Complete insurance at actuarial odds		Complete insurance at terms which make individual indifferent to buying no insurance	Complete insurance, at terms which range from those which make high-risk individual indifferent to buying insurance to terms which make low-risk individual indifferent to buying insurance. Firms may make loss on contract
Low risk	Complete insurance at actuarial odds	Partial insurance at actuarial odds	Complete insurance at terms which make individual indifferent to buying no insurance	Partial or no insurance; terms always such as to make individual indifferent to buying no insurance

4. CONTINUUM OF INDIVIDUALS

More interesting from an analytical point of view is the case where there is a continuum of individuals. We let $F(p)$ be the distribution function of individuals by their accident probability. Let $\underline{p}$ and $\bar{p}$ be the minimum and maximum accident probabilities respectively. We assume F is differentiable; f will denote the density function. We assume f is continuous. In a competitive market, we established that there was never an equilibrium. The behaviour of the monopolist is a fairly complex problem, but we can use many of the results of the previous section to give us some insights into its solution.

The " policy " set can be described by a function $\beta = \beta(\alpha)$ giving the maximum benefit available for any given premium. Clearly, these will be the only policies purchased. Thus, if

$$\begin{aligned} \alpha_1 &> \alpha_2, \\ \beta(\alpha_1) &> \beta(\alpha_2). \end{aligned} \qquad \ldots(4.1)$$

Let $\alpha(p)$ denote the policy purchased by individuals with accident probability p. It is clear that if

$$\begin{aligned} p_1 &> p_2, \\ \alpha(p_1) &\geqq \alpha(p_2). \end{aligned} \qquad \ldots(4.2)$$

It can also be shown that the policy function is a continuous function but it may not be differentiable. The points of non-differentiability are of some economic interest, for they are " pooling contracts ", i.e. contracts which are purchased by individuals with differing probabilities of having an accident (see fig. 7).

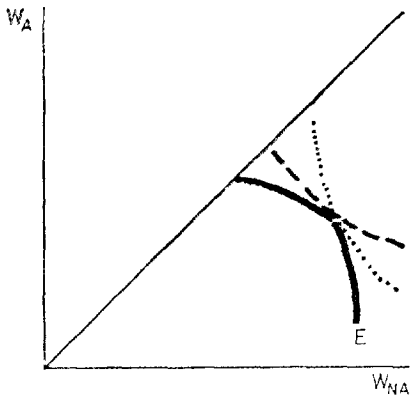

FIGURE 7

When β is a differentiable function of α,

$$\frac{d\beta}{d\alpha} = \frac{U'(W-\alpha)(1-p)}{U'(W-d+\beta)p} \equiv \lambda(\alpha, \beta, p) \qquad \ldots(4.3)$$

λ is the marginal rate of substitution of an individual with accident probability p, of income in state of nature " accident " for income in state of nature " no accident ". In this section, where there is no ambiguity, we shall write just W for W_0.

On the other hand, if at some $\{\hat{\alpha}, \beta(\hat{\alpha})\}$ β is not differentiable, but $d\beta^+/d\alpha$ and $d\beta^- d\alpha$ exist and $d\beta^+/d\alpha < d\beta^-/d\alpha$, then all individuals with

$$p^{**} < p < p^*,$$

where

$$\lambda(\hat{\alpha}, \hat{\beta}, p^{**}) = (\partial\beta/\partial\alpha)^+ \quad \text{and} \quad \lambda(\hat{\alpha}, \hat{\beta}, p^*) = (d\beta/d\alpha)^-$$

purchase the policy $\{\hat{\alpha}, \beta\}$.[3]

It immediately follows that for any piecewise differentiable policy function $\beta(\alpha)$,

$$\frac{d\beta}{dp} = \lambda(\alpha, \beta, p)\frac{d\alpha}{dp} \qquad \ldots(4.4)$$

(where it is understood that at points of non-differentiability we take, say, the left hand derivative).

Thus, we wish to choose $\{\alpha(p), \beta(p)\}$ functions, satisfying (4.4) to

$$\max \int \pi(p)f(p)dp. \qquad \ldots(4.5)$$

We analyse the problem using Pontryagin's technique. We form the Hamiltonian

$$H \equiv (\alpha(1-p)-\beta p)f + u\frac{d\alpha}{dp} + v\lambda\frac{d\alpha}{dp}$$

where α, β are state variables, and $d\alpha/dp$ is the control variable. Thus

$$\frac{d\alpha}{dp} = \left\{\text{indeterminate}\begin{matrix}\infty\\0\end{matrix}\right\} \quad \text{as} \quad u+v\lambda \gtreqless 0. \qquad \ldots(4.6)$$

We obtain the auxiliary equations

$$\frac{du}{dp} \equiv -\frac{\partial H}{\partial \alpha} = -(1-p)f - v\lambda_\alpha \frac{d\alpha}{dp} \qquad \ldots(4.7)$$

$$\frac{dv}{dp} = -\frac{\partial H}{d\beta} = pf - v\lambda_\beta \frac{d\alpha}{dp}. \qquad \ldots(4.8)$$

Our " strategy " for analysing the solution to this problem is

(i) If we can find a set of functions $\{\alpha(p), \beta(p), u(p), v(p)\}$ satisfying the equation

$$\frac{d\beta}{d\alpha} = \lambda(\alpha(p), \beta(p), p) \qquad \ldots(4.9)$$

and the equations (4.7) and (4.8), such that

$$u(p) + v(p)\lambda(\alpha(p), \beta(p), p) = 0$$

for all p *and*, letting $\tilde{p}$ denote the lowest p actually purchasing insurance, if

$$u(\bar{p}) = v(\bar{p}) = u(\tilde{p}) + \lambda v(\tilde{p}) = 0, \qquad \ldots(4.10a)$$

(the familiar transversality condition) and, (to ensure that the policies offered will be purchased),

$$V(\alpha(\tilde{p}), \beta(\tilde{p}), \tilde{p}) \geqq V(0, 0, \tilde{p}) \qquad \ldots(4.10b)$$

then $\{\alpha(p), \beta(p)\}$ gives the optimal set of policies. This is a solution which entails *complete separation*, i.e. every group buys a different policy.

(ii) It turns out, however, that there will not always exist such a solution. We then find a solution of the form

$$u(p) + v(p)\lambda(\alpha(p), \beta(p), p) \leqq 0, \quad \frac{d\alpha}{dp} \geqq 0 \qquad \ldots(4.11)$$

$$\{u(p) + v(p)\lambda(\alpha(p), \beta(p), p)\} \frac{d\alpha}{dp} = 0 \qquad \ldots(4.12)$$

if

$$u(p) + v(p)\lambda(\alpha(p), \beta(p), p) = 0, \quad \frac{d\beta}{d\alpha} = \lambda \qquad \ldots(4.13)$$

if

$$u(p) + v(p)\lambda(p) < 0 \quad \text{for} \quad p^{**} < p < p^*$$

$$\{\alpha(p), \beta(p)\} \quad \text{same for} \quad p^{**} < p < p^*$$

and

$$\left(\frac{d\beta}{d\alpha}\right)^+ < \lambda(\alpha(p), \beta(p), p) < \left(\frac{d\beta}{d\alpha}\right)^-$$

for which the functions satisfy (4.7) and (4.8) and the boundary values (4.10).

(4.11) says that $u + v\lambda$ is non-positive and $d\alpha/dp$ is non-negative. (4.12) says further that either $u + v\lambda = 0$ or $d\alpha/dp = 0$. (4.13) says that when $u + v\lambda = 0$, $d\beta/d\alpha = \lambda$, the marginal rate of substitution of the individual who purchases the policy; when $u + v\lambda < 0$, over an interval, all individuals in that interval purchase the same policy, and their marginal rates of substitution are bracketed by the left- and right-hand derivatives of $\beta(\alpha)$ at that " policy ".

Thus, whether equilibrium entails complete separation or some " pooling " (i.e. some

policy bought by individuals with different accident probabilities) depends on whether $u+v\lambda$ can be constant-at-zero for an interval of p. This in turn requires that

$$\frac{du}{dp}+\lambda\frac{dv}{dp}+v\left[\left(\lambda_\alpha+\lambda_\beta\frac{d\beta}{d\alpha}\right)\frac{d\alpha}{dp}+\lambda_p\right]$$

$$=(\lambda p-(1-p))f+v\lambda_p$$

$$=\left(\frac{U'(W-\alpha)}{U'(W-d+\beta)}-1\right)(1-p)f-v\frac{U'(W-\alpha)}{U'(W-d+\beta)p^2}=0 \qquad \ldots(4.14)$$

over an interval. Thus

$$v=\left(1-\frac{U'(W-d+\beta)}{U'(W-\alpha)}\right)h(p), \qquad \ldots(4.15)$$

where

$$h(p)\equiv(1-p)p^2f. \qquad \ldots(4.16)$$

Hence, we require (using (4.8) and (4.15))

$$\frac{dv}{dp}=h'\left(1-\frac{U'(W-d+\beta)}{U'(W-\alpha)}\right)-\frac{d\alpha}{dp}h\left[\frac{U''(W-d+\beta)}{U'(W-d+\beta)}\frac{(1-p)}{p}+\frac{U'(W-d+\beta)}{U'(W-\alpha)}\frac{U''(W-\alpha)}{U'(W-\alpha)}\right]$$

$$=pf\left[1+\frac{d\alpha}{dp}\left(1-\frac{U'(W-d+\beta)}{U'(W-\alpha)}\right)(1-p)^2\frac{U'(W-\alpha)}{U'(W-d+\beta)}\frac{U''(W-d+\beta)}{U'(W-d+\beta)}\right] \qquad \ldots(4.17)$$

i.e. letting $-U''(W)/U'(W)=A$

$$\frac{d\alpha}{dp}=\frac{pf+h'\left(\dfrac{U'(W-d+\beta)}{U'(W-\alpha)}-1\right)}{pf(1-p)\left[pA(W-\alpha)\dfrac{U'(W-d+\beta)}{U'(W-\alpha)}+(1-p)A(W-d+\beta)\dfrac{U'(W-\alpha)}{U'(W-d+\beta)}\right]}$$

$$=\phi(\alpha,\beta,p). \qquad \ldots(4.18)$$

The nature of the solution may now be characterized in terms of $(u+v\lambda)$. Let $u(\bar{p})=v(\bar{p})=0$, so at $p=\bar{p}$, $u+v\lambda=0$. Consider the backward solution. At $\bar{p}$, $\phi>0$ since we know $\beta-d=-\alpha$. Thus,

$$\left(\frac{d\alpha}{dp}\right)_{p=\bar{p}}=\frac{1}{(1-p)A(W-\alpha)}>0 \quad \text{if} \quad A<\infty.$$

There is never pooling among the most high risk individuals.

(Actually, all the above argument has established is that complete separation is always consistent with the Pontryagin equations for accident probabilities near $\bar{p}$. To see that pooling is not feasible, observe that at $\bar{p}$, using (4.10), (4.7) and (4.8)

$$\frac{d(u+v\lambda)}{dp}=f(1-\bar{p})\left(\frac{U'(W-\alpha)}{U'(W-d+\beta)}-1\right)-\frac{U'(W-\alpha)v}{U'(W-d+\beta)p^2}\leqq 0$$

if $\alpha<d-\beta$, in which case $u+\lambda v>0$ for p near $\bar{p}$, which is inconsistent with pooling; if $\alpha=d-\beta$, $d^2(u+v\lambda)/dp^2=-(1/\bar{p}^2)dv/dp=-f/p<0$ which is again consistent with pooling.

Since there is no pooling near $\bar{p}$,

$$\left.\frac{d(u+\lambda v)}{dp}\right|_{\bar{p}}=0, \quad \text{implying} \quad v(\bar{p})=\left(1-\frac{U'(W-d+\beta)}{U'(W-\alpha)}\right)h(p)$$

Since $v(\bar{p})=0$, this implies that $\beta(\bar{p})=d-\alpha(\bar{p})$, provided $f(\bar{p})>0$.

We now consider the case where $h'>0$. Then $d\alpha/dp>0$ everywhere. We simply solve the equations

$$\frac{d\alpha}{dp} = \phi(\alpha, \beta, p)$$
$$\frac{d\beta}{dp} = \phi(\alpha, \beta, p)\lambda(\alpha, \beta, p) \qquad \ldots(4.19)$$

with the boundary values

$$V(\alpha(\tilde{p}), \beta(\tilde{p}), \tilde{p}) = V(0, 0, \tilde{p}) \qquad \ldots(4.20)$$

and

$$\alpha(\bar{p}) = d-\beta(\bar{p}).$$

Thus, if $h'>0$ there is no pooling. (There may be an interval $\underline{p} \leqq p<\tilde{p}$ in which no one purchases insurance.)

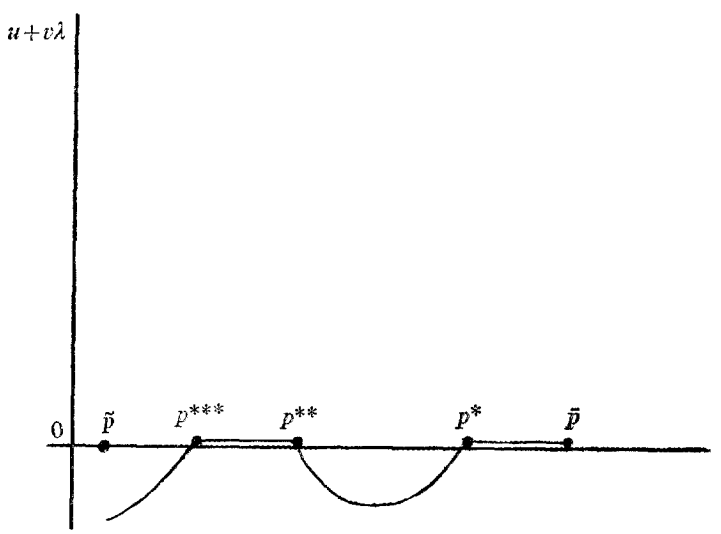

FIGURE 8

If $h'<0$, there may exist a p^* such that $\phi(\alpha, \beta, p)<0$. Then, for $p^{**}<p<p^*$, everyone buys the same policy; p^{**} is the point such that

$$u(p^*)+\lambda(\alpha, \beta, p^*)v(p^*) = u(p^{**})+\lambda(\alpha, \beta, p^{**})v(p^{**}) = 0. \qquad \ldots(4.21)$$

In Figure 8 we have depicted a case where there is another " pooling " contract, which is purchased by individuals with

$$p<p^{***}.$$

Returning to the definition of h, we observe that

$$h' = (-3p^2+2p)f+(1-p)p^2f'. \qquad \ldots(4.22)$$

Thus

$$h'>0 \quad \text{if} \quad \frac{f'}{f} > \frac{3p-2}{p(1-p)}. \qquad \ldots(4.23)$$

Clearly, if

$$\bar{p}<\tfrac{2}{3} \quad \text{and} \quad f' \geqq 0,\ h'>0. \qquad \ldots(4.24)$$

But in general, $h'<0$ for some values, of p, so there *may be* pooling.

This is as much as we have been able to extract from the general theory. Some examples may help illuminate the structure of the solution.

Example 1. *Locally risk neutral individuals*

We let

$$\mathscr{W} = pW_A + (1-p)W_{NA} \quad \text{if} \quad W_A > \delta W_{NA}, \quad \delta < 1 \qquad \ldots(4.25)$$

$$\mathscr{W} = W_A\left(p + \frac{(1-p)}{\delta}\right) \quad \text{if} \quad W_A < \delta W_{NA}. \qquad \ldots(4.26)$$

It is easy to establish that the only insurance policy entails complete insurance, i.e.

$$\alpha = d - \beta. \qquad \ldots(4.27)$$

This will be purchased by all individuals for which

$$W_0 - \alpha \geqq (W_0 - d)\left(p + \frac{(1-p)}{\delta}\right)$$

i.e.

$$p \geqq \frac{1}{1-\delta}\left(1 - \left(\frac{W_0 - \alpha}{W_0 - d}\right)\delta\right) \equiv p^*(\alpha). \qquad \ldots(4.28)$$

Profits are then

$$\alpha(1 - F(p^*)) - d\int_{p^*}^{p_{\max}} pf(p)dp \qquad \ldots(4.29)$$

which are maximized when

$$1 - F(p^*) = \frac{(\alpha - dp^*)f(p^*)\delta}{(1-\delta)(W_0 - d)}. \qquad \ldots(4.30)$$

Example 2

If α is a linear function at β, and $-U''/U'$ is constant then the density function will be of the form

$$f = \frac{c}{p^{\frac{1+2\lambda}{1+\lambda}}(1-p)^{\frac{2+\lambda}{1+\lambda}}}, \qquad \ldots(4.31)$$

where

$$\lambda = \frac{1 - p_{\max}}{p_{\max}}.$$

Proof. We assume

$$\frac{d\beta}{d\alpha} = \frac{U'(W-\alpha)(1-p)}{U'(W-d+\beta)p} = \lambda = \text{constant}, \qquad \ldots(4.32)$$

i.e.

$$\frac{d\lambda}{dp} = \left(\frac{\partial\lambda}{\partial\alpha} + \frac{\partial\lambda}{\partial\beta}\lambda\right)\frac{d\alpha}{dp} + \frac{\partial\lambda}{\partial p}$$

$$= \lambda[A(W-\alpha) + \lambda A(W-d+\beta)]\frac{d\alpha}{dp} - \frac{\lambda}{p(1-p)} = 0. \qquad \ldots(4.33)$$

$$\frac{d\alpha}{dp} = \frac{1}{A(1+\lambda)p(1-p)}. \qquad \ldots(4.34)$$

But from (4.18)

$$\frac{d\alpha}{dp} = \frac{h + h'\left(\frac{(1-p)^2}{\lambda} - p(1-p)\right)}{h(1-p)A\left[\frac{1-p}{\lambda} + p\lambda\right]}. \qquad \ldots(4.35)$$

Equating (4.34) and (4.35) we obtain, after simplification,

$$\frac{h'}{h} = \frac{1}{(1+\lambda)(1-p)p}.$$

Upon integration, and using (4.16) we obtain (4.31). Similarly restrictive conditions obtain for other utility functions.

The main results of this section may be summarized as follows:

(1) The monopolist will always offer a continuum of contracts, provided individuals are unambiguously risk averse. (Example 1 provided an example where individuals were not averse to small risks.)

(2) The premium may be either a convex or concave function of the benefits. Only in special cases will it be linear (Example 2).

(3) Whereas, in competitive markets, equilibrium was characterized by different individuals purchasing different contracts (i.e. no pooling of risks), with monopoly the same policy may be purchased by individuals of different risks. A sufficient condition for " separation " at p, i.e. every type of individual with accident probability " near " p purchasing a different contract, is that the density of function at p satisfy

$$\frac{f'}{f} > \frac{3p-2}{p(1-p)}.$$

(4) There may be a set of individuals (those with the lowest accident probabilities) who do not purchase any insurance.

5. TWO GROUPS DIFFERENT IN ATTITUDES TOWARDS RISK

In a competitive economy, if individuals differed only in attitudes towards risk then risk neutral competitive insurance firms would offer complete insurance and would pay no attention to the degree of risk aversion of a particular individual. Thus, the absence of information about attitudes towards risk of particular individuals is of no importance. For a monopolist, this information is, however, valuable. If the monopolist knew which individuals were highly risk averse and which were only mildly risk averse, it would act as a discriminating monopolist, offering each complete insurance, but at different terms.

On the other hand, if it does not know who is highly risk averse and who is not, it can obtain this information by offering the individual a choice from among a set of contracts. But since it obtains a smaller profit from partial insurance than from complete insurance, it must trade off the gains from the ability to discriminate against the loss in profits from selling only partial insurance to the not very risk averse. Thus, in the case with two groups, the optimal set of policies sold may either entail a single contract, purchased only by the very risk averse, a single contract purchased by everyone, or two contracts.

By exactly the same kinds of arguments that we used earlier, we can establish

Property 5. *The high risk aversion group obtains complete insurance.*

Property 6. *If the low risk aversion group purchases insurance, it purchases a policy which leaves it at approximately the same level of expected utility as it had without insurance.*

More precisely, let $\{\alpha_L, \beta_L\}$ and $\{\alpha_H, \beta_H\}$ denote the policies purchased by the low risk aversion and high risk aversion groups respectively. Then

$$\alpha_H = d - \beta_H \qquad \ldots(5.1)$$

$$V(0, 0, R_L) = V(\alpha_L, \beta_L, R_L) \geqq V(\alpha_H, \beta_H, R_L) \qquad \ldots(5.2)$$

$$V(\alpha_H, \beta_H, R_H) = V(\alpha_L, \beta_L, R_H), \qquad \ldots(5.3)$$

where $V(\alpha, \beta, R_H)$ is the expected utility of the high risk aversion individual who obtains policy $\{\alpha, \beta\}$; similarly for $V(\alpha, \beta, R_L)$. Profits are given by

$$\bar{\pi} = N_R(\alpha_H - dp) + N_L(\alpha_L(1-p) - \beta_L p).$$

From (5.1-5.3), we immediately obtain the result that

$$\frac{d\bar{\pi}}{d\alpha_H} = N_R - \frac{N_L(1-\gamma_L)\gamma_H}{\gamma_L - \gamma_H} \frac{V_1(\alpha_H, \beta_H, R_H)}{V_1(\alpha_L, \beta_L, R_H)} \qquad \ldots(5.4)$$

where

$$\gamma_i = -\frac{V_1(\alpha_L, \beta_L, R_i)}{V_2(\alpha_L, \beta_L, R_i)} \frac{p}{1-p}.$$

Thus, if

$$\frac{N_R}{N_L} > \frac{(1-\gamma_L(0, 0, R_L))\gamma_H(0, 0, R_H)}{\gamma_L(0, 0, R_L) - \gamma_H(0, 0, R_H)} \frac{V_1(\alpha_H, \beta_H, R_H)}{V_1(0, 0, R_H)}$$

only one kind of policy is purchased.

Consider now the limit as α_H increases and $\{\alpha_L, \beta_L\} \to \{\alpha_H, \beta_H\}$, $\gamma_L \to 1$ and $\gamma_H \to \gamma_L$. Hence, to evaluate the second term of (5.4) we use L'Hopital's rule to obtain

$$\lim \frac{(1-\gamma_L)}{\gamma_L - \gamma_H} = \frac{-A_L}{A_L - A_H},$$

where

$$-A_i = \frac{U_i''(W-\alpha)}{U_i'(W-\alpha)}, \quad i = R, L$$

(where the subscript refers to the risk aversion of the group), the Arrow–Pratt measure of absolute risk aversion. Hence if $N_R/N_L < A_L/(A_H - A_L)$ then only a single policy, involving complete insurance, will be offered.

In short, we have established

Property 7. *There is a critical ratio of highly risk averse to low risk aversion individuals, such that if the ratio in the population exceeds the critical ratio, only the risk averse purchase policies. There is another critical ratio, such that if the ratio in the population is less than this, there is only one policy, purchased by both groups (entailing complete insurance). In between two policies are offered, complete insurance for the high risk averse, partial insurance for the less risk averse.*

6. CONCLUDING COMMENTS

This paper has attempted to show that the scope for at least partial discrimination by a monopolist among his customers is much greater than has previously been thought. The attempt to discriminate among his customers leads the monopolist to engage in practices restricting the set of insurance contracts he sells, using non-linear price schedules, bundling and tie-in sales, random prices (see Salop), which would otherwise be difficult to explain. Some of these practices, such as the use of non-linear pricing, would occur in some circumstances in competitive markets, e.g. in insurance markets, under imperfect information; most of them, however, would not. Thus monopoly and competition differ in far more significant ways than just simply the price charged. Whether these practices may occur in market structures other than the polar ones analysed here is a question which will be pursued elsewhere.

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NOTES

1. This follows from the fact that

$$\lim_{\theta \to 0} \frac{\mu(\theta)}{f(\theta)} = \lim_{\theta \to \infty} \frac{\mu(\theta)}{f(\theta)} = 0.$$

The problem in establishing this arises because, by the transversality condition (provided $\underline{x}$ is positive)

$$\lim_{\theta \to 0} \mu(\theta) = \lim_{\theta \to \infty} \mu(\theta) = 0,$$

while for distributions without finite support

$$\lim_{\theta \to 0} f(\theta) = \lim_{\theta \to \infty} f(\theta) = 0.$$

Hence we use L'Hopital's rule to establish

$$\lim \frac{\mu}{f} = \begin{cases} \lim \dfrac{f - \mu U_{00}/U_0}{f'} = 0 & \text{if } f' > 0 \text{ and } U_{00}/U_0 \text{ is bounded} \\ \lim \dfrac{\partial^{n-1} f/\partial\theta^{n-1}}{\partial^n f/\partial\theta^n} = 0 & \text{if } \partial^n f/\partial\theta^n \text{ is the first non-zero derivative.} \end{cases}$$

(1.7)-(1.9) were derived on the hypothesis that all individuals consume all commodities. If this is not true the equations have to be modified. In particular it is possible that $R_i(0) > 1$.

In some other problems, for certain distributions without a finite support, $\lim_{\underline{x} \to \infty} R_i(\underline{x}) > 1$, as the next example illustrates.

2. Points above the 45° line represent in effect more than 100 per cent insurance: after buying the insurance policy, the individual is actually better off in the state of nature " accident " than he is in the state of nature " no accident ". Although such contracts are clearly conceivable, they will not be observed in their monopolistic or competitive markets, and so we shall ignore them. The points to the north-east of E represent a negative premium with a positive benefit, to the south-east a negative benefit with a negative premium, and to the south-west of E a positive premium with a negative benefit.

3. That is, the individual maximizes $U(W-\alpha)(1-p) + U(W-d+p)p$ so if $d\beta/d\alpha$ exists,

$$-U'(1-p) + U'p\frac{d\beta}{d\alpha} = 0.$$

If $(d\beta/d\alpha)^+$ and $(d\beta/d\alpha)^-$ exist, then clearly there exists an interval $p^{**} < p < p^*$ such that

$$-U'(1-p) + pU'\left(\frac{d\beta}{d\alpha}\right)^+ < 0$$

$$-U'(1-p) + pU'\left(\frac{d\beta}{d\alpha}\right)^- > 0$$

i.e. all individuals in the interval buy the same policy.

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