Between Search and Walras

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We present a model in which unemployed workers simultaneously sample *n* potential employers. By varying *n*, we nest search and Walrasian-type models of the labor market. We show that low values of *n* yield typical search equilibria: the wages are dispersed below the marginal productivity of labor. Interestingly, as *n* exceeds a relatively small threshold, the Walrasian-type equilibrium emerges with the competitive wage quoted by all firms. For intermediate values of *n*, the equilibrium is a hybrid of the Walrasian and search equilibria. The model generates wage rigidity and yields novel predictions regarding the comovement of wages, firm turnover, and unemployment.

> "Stop Wasting Time Sending Too Few Resumes" (From a frequent ad in the WALL STREET JOURNAL)

I. Introduction

Two dominant approaches to modeling the labor market are found in the literature. The Walrasian paradigm endows workers and firms with perfect information about the labor market conditions; in particular, every worker is assumed to know the wage offered by every firm in the economy. At the other extreme is the search literature, which generally assumes that job-seeking workers receive offers sequentially, one at a time. Two

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basic features of the Walrasian paradigm are (*a*) full employment and (*b*) equality between wages and the marginal product of labor. Search models allow for "frictional" unemployment and typically yield wages below the marginal product of labor.[1] In this article we present a framework that nests both search and Walrasian-type models and study the evolution of stationary equilibria in the largely unexplored region between the two approaches.

In our model all workers are equally productive. The product market is perfectly competitive with free entry and exit of firms. To model the labor market, we apply a modified version of Burdett and Judd's (1983) "noisy search": an unemployed worker can simultaneously approach $n \geq 2$ randomly selected firms every period.[2] An important feature of our model is that some firms extend an offer, while others have no vacancies; applying workers cannot distinguish between them. If the applicant receives at least one wage offer, he may accept it and commence employment immediately; otherwise, he must wait one period to approach another n firms until he receives an acceptable offer. While the number of applications sent by the worker in each period is exogenously set to n, the number of wage offers received is determined in equilibrium. We regard n as a proxy for information completeness: the more firms are sampled by the worker in each period, the better informed he is about the labor market conditions. In the limit, as workers observe wage offers made by every firm in the economy, our model converges to a Walrasian model.

We believe that multiple contemporaneous search opportunities fit the reality of many labor markets. Unemployed workers should not and do not feel compelled to seek employment with only one firm at a time, waiting for the outcome before approaching other firms. Graduating students enter the job market by sending resumes to many potential employers simultaneously. Placement firms offer their clients an opportunity to search simultaneously for multiple job openings. Accordingly, our model pertains to situations in which job candidates can simultaneously apply to several firms.

The main result of the article is the characterization of equilibria for various levels of search intensity. We show that low levels of search intensity yield an equilibrium characterized by wage dispersion below the marginal product of labor, as in typical search models. As n increases, we observe a gradual transition of the equilibrium wage structure from the search type to the Walrasian type. When the number of sampled firms becomes sufficiently large, all firms offer a wage equal to the marginal product of labor:

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[1] See, e.g., Diamond (1971, 1982), Wilde (1977), Albrecht and Axell (1984), Albrecht and Jovanovic (1986), Burdett and Mortensen (1989), Pissarides (1990), and Chalkley (1991).

[2] The basic model goes back to the seminal paper by Stigler (1961).

the competitive wage. The intuition behind the result is that an increase in search intensity puts a competitive pressure on hiring firms by increasing the worker's information set and employment opportunities.

We compute the equilibria of our model for an array of parameter values and find that the competitive equilibrium prevails at relatively low levels of search intensity; for example, $n = 9$ for a reasonable choice of parameter values. For the same parameter values, $n = 7$ and $n = 8$ yield a nondegenerate wage distribution below the competitive wage with a mass point at the competitive wage. Finally, at $n \leq 6$ the wage distribution is everywhere below the competitive wage, as in a typical search model. In all three cases the model yields unemployment, which declines with n. These results justify the use of a competitive framework to study labor markets in which workers can sample a moderate number of firms simultaneously, even though their information about the labor market is far from perfect. Conversely, the simple sequential search model is shown to be representative of labor markets in which workers can sample only a few firms simultaneously. This distinction offers sharper predictions for future empirical work.

Our model exhibits intriguing aggregate labor market implications. For example, technology shocks affect wages, but not employment, whereas changes in the rate of firm mortality as well as changes in search technology cause changes in employment, but do not affect wages. The reason is that employment is determined by the probability of a match between workers and firms (search technology) and by the rate of job creation/destruction driven by firm mortality. Technology shocks affect only wages and firm size, but not the aggregate employment level. This dichotomy allows for large variations in employment to be associated with relatively small shifts in wages, resulting in the appearance of wage rigidity. While this dichotomy is driven by simplifying assumptions, we argue that in a more general setup large variations in employment are associated with relatively small shifts in wages, resulting in the appearance of wage rigidity.

Several articles study models with variable search intensity. Butters (1977), Wilde (1977), Burdett and Vishwanath (1988), Lang (1991), Montgomery (1991), and Acemoglu and Shimer (1997) study models in which agents choose their search intensity. Typically, these models yield wage dispersion. The intuition behind this dispersion is that without it agents have no reason to search, thus the equilibrium collapses to that of a monopsony, that is, Diamond-type equilibrium.[3] In our model, unemployed workers apply to randomly chosen firms, some of which—the

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[3] An exception is Burdett and Vishwanath (1988), who allow search on the job and assume that the marginal cost of search approaches zero at low levels of search. Consequently, they find that all workers receive wages equal to the marginal product of labor.

new entrants—have vacancies, while others do not hire. Consequently, even if all the hiring firms offer the same wage, searching workers still face a heterogeneous population of firms.[4] Thus, higher search intensity (higher n) always raises the expected wage by increasing the probability of encountering a hiring firm. Therefore, if workers were free to choose their search intensity, sufficiently low search cost would rule out Diamond-type equilibrium.[5]

The article is organized as follows: in Section II we set up the model; in Section III we characterize stationary equilibrium, derive necessary and sufficient conditions for its existence, and prove its existence. Section III also contains simulation results for a specific production function. In Section IV we discuss implications of the model for aggregate employment and wage determination. Section V concludes the article.

II. Model

We postulate a discrete-time infinite-horizon model of an economy with a continuum of workers and firms. Firms produce and sell an identical good in a perfectly competitive market. Labor is the only variable input. Unemployed workers find jobs through a process of search specified below.

Setup

In each period, the following sequence of events takes place: first, new firms enter the market and stand ready to hire workers. Second, every unemployed worker sends n applications to randomly chosen firms, including the new entrants. Third, all hiring firms send binding job offers to some or all of their applicants. Fourth, unemployed workers receive the offers and make a decision either to become employed or to remain unemployed and search again next period. Fifth, production takes place, and wages are paid. Finally, some firms go out of business and lay off their workers, who join the ranks of the unemployed and search during the next period.

The main feature of our model is that unemployed workers simultaneously solicit job offers from $n \geq 2$ randomly selected firms. A worker can choose to accept the highest wage offered to him or to stay unemployed and search again in the next period, in which case he forgoes the wage during the current period. If a worker receives several identical desirable

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[4] Vroman (1985) also assumes that workers may encounter zero wage offers, but workers in her model may sample only one offer per period.

[5] We were able to show that for every equilibrium derived in our model there is a class of well-behaved cost-of-search functions, which yields the same equilibrium in an economy with endogenous search intensity. These results are available from the authors on request.

offers, he chooses one at random, giving equal probability to each offer. We assume that employed workers cannot search. Firms cannot search, either, and can only extend offers to applying workers.

Firms

All firms use the same technology. In order to produce, each firm has to spend a fixed amount K every period, which is paid upon entry at the very beginning of the period. Once the firm spent K, its output is given by $F(l)$, where l is the measure of workers it employs. We assume that $F(\cdot)$ is strictly increasing, concave, twice continuously differentiable, $F(0) < K$, and there exists $l < \infty$, such that $F(l) > K$. Since the product market is competitive, all firms charge the same price, which is normalized to be one.

There is free entry of firms: in the equilibrium, the measure of firms is such that each firm earns zero profits. In every period, an exogenously given proportion θ of firms suffer a negative shock, stop producing, and lay off their workers. The exit of firms tends to raise profits of the remaining firms, which causes new firms to enter and hire unemployed workers. To simplify the analysis, we assume that each firm hires new workers only upon entering and cannot hire new workers thereafter. It also has to offer the same wage to all new hires.[6]

Each firm chooses how many offers to make, O, and the wage rate associated with these offers, w. Recall that offers can be made only to applying workers. An offer constitutes a commitment on the part of the firm to hire a worker who accepts it. Each firm takes into account that some workers will decline its offers if they get better or identical offers. We denote by x the measure of applicants per firm, and by $e(w)$ the proportion of workers accepting the extended offer w; both are determined endogenously.[7]

The firm's optimization problem is to maximize the per-period profit under the constraint that it can make offers only to the applicants and that some of its offers will be declined:

$$\max_{O,w} F(l) - wl - K$$

$$\text{subject to } O \le x;$$

$$l = Oe(w).$$

We denote by $G(w)$ the equilibrium proportion of firms that offer a

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[6] Without this simplifying assumption, firms in our model would be able to hire workers over several periods at varying wage rates. In such a case we would have to take into account the entire history of each firm. This would tremendously increase the complexity of the model.

[7] As in Burdett and Judd (1983), we treat x as deterministic.

wage smaller than or equal to w. For notational simplicity, we treat firms with no vacancies as if they offer a wage of zero.

Workers

There is a continuum of workers in the economy: we normalize their measure to be one. Let L denote the measure of aggregate employment and $U \equiv 1 - L$ denote the measure of unemployed workers. Unemployed workers seek to maximize the expected discounted value of their lifetime wage earnings, denoted by V. Let $V(w)$ be the expected discounted lifetime wage of an unemployed worker who has just received n offers, of which w is the highest, and has to decide whether to accept it or to continue searching during the next period. Thus,

$$V(w) = \max\{w + (1-\theta)\beta V(w) + \theta\beta V, \beta V\}.$$

Hence, a worker accepts employment if $w + (1-\theta)\beta V(w) \geq (1-\theta)\beta V$ and otherwise remains unemployed and searches again the following period. This implies the existence of a reservation wage $\underline{w}$ satisfying the equation

$$\underline{w} + (1-\theta)\beta V(\underline{w}) = (1-\theta)\beta V.$$

A worker accepts employment if and only if his highest current offer is greater than or equal to $\underline{w}$. It should be noted that even if all the hiring firms offer a wage greater than $\underline{w}$, workers who happened to sample only nonhiring firms (i.e., firms that entered before this period) remain unemployed.

III. Stationary Equilibrium

Definition. Given the parameters n, θ, β, K, and the production function $F(l)$, an equilibrium in this model is represented by a stationary distribution of wage offers $G(w)$, an acceptance function $e(w)$, and the aggregate employment level L such that

1. whenever a worker is unemployed, he searches for a job and chooses whether to accept or reject the highest offer made during the period so as to maximize his expected lifetime discounted wage earnings, given $G(w)$;
2. given $e(w)$, every firm maximizes its profits by choosing the number of offers O and the wage w. The resulting wage offers distribution is $G(w)$;
3. given $G(w)$, a worker receiving an offer w accepts it with probability $e(w)$;
4. each existing firm earns zero profit, and every firm that can earn

zero profit enters the economy;[8]

5. the aggregate employment, L, stays constant over time.

We now characterize the equilibrium in the general case.

General Case

In equilibrium all operating firms make zero profits:

$$F(l) - wl - K = 0.$$

Let $w(l)$ define the wage that a firm employing l workers has to pay to earn zero profit. Thus, $w(l)$ satisfies

$$w(l) = \frac{F(l) - K}{l}. \tag{1}$$

Given our assumptions about $F(l)$, the function $w(l)$ is well defined for positive l, single peaked, and twice continuously differentiable. Let l^* be the level of employment at which $w(l)$ attains its maximum, and let $w^* \equiv w(l^*)$. It follows that

$$F'(l^*) = w^*.$$

In the Walrasian model w^* and l^* are the equilibrium wage and firm size (hereafter we refer to them as the competitive wage and employment/firm size, respectively). Since $w'(l) > 0 \; \forall l < l^*$, we can invert $w(l)$ in this range. We denote the resulting function by $l(w)$ over $w \in (0, w^*)$; it represents the measure of workers employed by a firm offering w and making zero profits. From the properties of $w(l)$, it follows that for every $w \in (0, w^*)$, $l(w)$ is monotonically increasing and twice continuously differentiable. The zero-profits condition implies that every firm offering w must employ exactly $l(w)$ workers in the equilibrium.

Recall that θ is the steady state proportion of firms that exit the economy every period. In a stationary equilibrium, θ is also the steady state proportion of firms that enter. The equilibrium number of workers laid off every period equals the number of workers who find a job during the same period, such that there is no change in aggregate employment. We assume that the probability that a firm suffers a negative shock is independent of the wage it pays, which implies that θ is also the steady state proportion of workers who are hired and laid off each period.

We denote by $\rho \in [0, \theta]$ the equilibrium proportion of firms offering the competitive wage w^*, and by $\overline{w}$ the highest wage offer among firms that do not offer w^*. Recall that $\underline{w}$ is the reservation wage of searching

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[8] The assumption that all firms that can make zero profits do enter simplifies the analyses by ruling out equilibria in which firms get more applicants than the number of offers they extend (see claim 3 below). This simplifies the model, yet it does not change the results qualitatively.

workers. Next, we characterize the equilibrium distributions $G(w)$ and $e(w)$ by proving a set of claims that spell out the mechanics of the model.

Claim 1 shows that the wage distribution function is continuous everywhere except, perhaps, at the competitive wage. It rules out a Diamond-type equilibrium, in which all hiring firms offer the monopsony wage $\underline{w}$.

CLAIM 1. If $G(w)$ has a mass point at some wage, $w' > 0$, then $w' = w^*$.

Proof. Suppose that $w' < w^*$. Then by infinitesimally raising its wage offer a firm can attract all its applicants who received another offer of w' and who would have otherwise chosen one firm at random. Since $G(w')$ is a mass point, there is a nonzero measure of such workers. However, since $l(w') < l(w^*)$, and $F'' < 0$, it follows that $F'[l(w')] > w'$. Therefore, by infinitesimally raising its offer, a firm can recruit additional workers whose marginal product exceeds their wage. Its profit would rise, which cannot be in equilibrium.

Claim 2 follows directly from claim 1. It simply states that a worker accepts a wage below w^* only if he was not offered a higher wage in his other $n - 1$ draws. As we show later, this statement is not necessarily true when he encounters several offers of w^* and randomizes among them.

CLAIM 2. For every $w \in [\underline{w}, w^*)$,

$$e(w) = G(w)^{n-1}.$$

Proof. By claim 1, $G(w)$ has no mass point at any $w < w^*$. Therefore, a firm that offers a wage below the competitive wage can only hire workers who did not receive a higher offer in their other $n - 1$ draws. The probability of such event is $G(w)^{n-1}$, and, therefore, for $\underline{w} \leq w < w^*$, $e(w) = G(w)^{n-1}$.

Claim 3 states that every hiring firm makes offers to all its applicants.

CLAIM 3. Every hiring firm makes offers to all applicants, regardless of the wage it offers, that is, $O = x \; \forall w \geq \underline{w}$.

Proof. First consider a firm offering $w \in [\underline{w}, \overline{w}]$. Suppose this firm makes offers only to some applicants, $O < x$, and thus recruits $Oe(w)$ workers. By claims 1 and 2, $e(w)$ is continuous in the range $w \in [\underline{w}, \overline{w}]$. Hence, this firm could attract the same number of workers, while reducing its wage bill, by raising O and reducing w, holding $Oe(w)$ constant. This way the firm can reduce its wage bill without reducing its output, which cannot be in equilibrium. Thus, firms offering $w \in [\underline{w}, \overline{w}]$ make offers to all applicants.

Now consider an entering firm offering w^*, and suppose that $O < x$. An identical firm can enter and make zero profits by attracting workers who received no other offers. Such entry reduces $e(w^*)$ infinitesimally. However, since $O < x$, the first firm can continue making zero profits by slightly raising O. This contradicts the equilibrium condition that all firms that can make zero profits enter. Therefore, in equilibrium, firms offering w^* must also make offers to all applicants.

Claim 4 shows that the distribution of offers has a connected support, except perhaps between $\overline{w}$ and w^*.

CLAIM 4. For every $w \in [\underline{w},\overline{w}]$, $G'(w) > 0$.

Proof. Suppose not. Then there are two firms offering w_1 and w_2, such that $w_1 < w_2$ and $G(w_1) = G(w_2) > 0$. Hence, both firms recruit the same number of workers, implying that the firm that pays w_1 earns strictly higher profits, which contradicts the zero-profit constraint.

Claim 5 derives the worker's acceptance rate for a firm offering the competitive wage.

CLAIM 5.

$$e(w^*) = \frac{[1-(1-\rho)^n]}{\rho n}.$$

Proof. All applicants are the same ex ante. However, after observing all their offers, applicants at a firm offering w^* can be separated into n types according to the number of offers of w^* they received. Those who received i other offers, $i \in \{0, 1, 2, \ldots, n-1\}$, in addition to the one extended by this firm, accept its offer with probability $\frac{1}{i+1}$. Consequently, the proportion of workers that choose this firm's offer is

$$e(w^*) = \sum_{i=0}^{n-1} \frac{1}{i+1}\binom{n-1}{i}\rho^i(1-\rho)^{n-i-1}.$$

Notice that

$$\begin{aligned}\frac{1}{i+1}\binom{n-1}{i} &= \frac{(n-1)!}{(i+1)i!(n-i-1)!}\\ &= \frac{n!}{n(i+1)!(n-i-1)!} = \frac{1}{n}\binom{n}{i+1}.\end{aligned}$$

Substituting

$$\frac{1}{i+1}\binom{n-1}{i} = \frac{1}{n}\binom{n}{i+1},$$

multiplying by ρ/ρ, and letting $j \equiv i+1$, we can rewrite $e(w^*)$ as

$$e(w^*) = \frac{1}{n\rho}\sum_{j=1}^{n}\binom{n}{j}\rho^j(1-\rho)^{n-j}.$$

Noting that

$$\sum\nolimits_{j=0}^{n}\binom{n}{j}\rho^{j}(1-\rho)^{n-j} = 1,$$

we obtain the result

$$e(w^{*}) = \frac{1-(1-\rho)^{n}}{n\rho}.$$

Claim 6. If $\rho > 0$, then $\overline{w} < w^{*}$.

Proof. Suppose $\overline{w} = w^{*}$. Then by claim 4 there exists a firm offering a wage, $w = w^{*} - \epsilon$, where $\epsilon > 0$ is infinitely small. None of the applicants to this firm who were also offered w^{*} will accept its offer; thus, it can hire only $x(1-\rho)^{n-1} << l^{*}$ workers. This implies that even an infinitesimal wage reduction from w^{*} yields a relatively large decline in attracted labor, regardless of the deviation size. Since the marginal product of all the lost workers exceeds their wage, a firm offering $w = w^{*} - \epsilon$ makes negative profits. Consequently, $\overline{w} < w^{*}$.

The last claim states that the measure of workers the firm employs (those that accept its offer) in equilibrium has to equal the measure of workers satisfying the zero-profit constraint $l(w)$.

Claim 7. For wage offers, $w \in [\underline{w},\overline{w}]$, $l(w) = x[G(w)]^{n-1}$.

Proof. Firms offering wage $w \in [\underline{w},\overline{w}]$ can only hire workers who were offered less in their other $n-1$ draws. Thus, the measure of workers hired by such a firm equals the measure of applicants multiplied by the probability of their acceptance of the offer: $x[G(w)]^{n-1}$. The zero-profit condition implies that the measure of hired workers has to equal $l(w)$; it follows that $l(w) = x[G(w)]^{n-1}$.

Proposition 1 follows immediately from claims 1–7.

Proposition 1. In the steady state for $n \geq 2$,[9]

i)

$$G(w) = \begin{cases} 1-\theta & \text{if } w < \underline{w} \\ \left[\dfrac{l(w)}{x}\right] & \text{if } \underline{w} \leq w < \overline{w} \\ 1-\rho & \text{if } \overline{w} \leq w < w^{*} \\ 1 & \text{if } w \geq w^{*} \end{cases}. \qquad (2)$$

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[9] If $n = 1$, our model yields the standard Diamond paradox: $\underline{w} = 0$, and $G(\underline{w}) = 1$.

ii)

$$e(w) = \begin{cases} 0 & \text{if } w < \underline{w} \\ G(w)^{n-1} & \text{if } \underline{w} \leq w < w^* \\ \dfrac{1-(1-\rho)^n}{\rho n} & \text{if } w = w^* \\ 1 & \text{if } w > w^* \end{cases} . \tag{3}$$

iii) All firms make offers to all the applying workers.

Claim 3 tells us that firms will make offers to all the applicants. Moreover, given the equilibrium acceptance function, $e(w)$, a firm is indifferent between any wage in the range $[\underline{w},\overline{w}]$ and, if $\rho > 0$, w^*, as it makes zero profit throughout. We have also shown that the firm will not choose $w < \underline{w}$ or $w \in [\overline{w},w^*)$ since it loses money in these wage ranges. Consequently, we know that all firms maximize their profits in this equilibrium.

Proposition 1 identifies three types of equilibrium wage distributions.

Type 1—Search Equilibrium. The equilibrium distribution of wages has no mass point: $\rho = 0$. In this case $G(w)$ is continuous on $w \in [\underline{w},\overline{w}]$, where $\overline{w} \leq w^*$. This equilibrium exhibits typical features of a search model.

Type 2—Walrasian Equilibrium. All firms offer the competitive wage $\rho = \theta$. This corresponds to the Walrasian equilibrium, with one important distinction: a proportion of workers remains unemployed every period.

Type 3—Mixed Equilibrium. The equilibrium distribution combines the features of a Walrasian model with those of a search equilibrium. It has a mass point at w^*, however, $\rho < \theta$, implying that only some hiring firms offer the competitive wage, while the rest offer lower wages distributed on $[\underline{w},\overline{w}]$, where $\overline{w} < w^*$.

It is helpful to illustrate the role of our assumptions by contrasting the features of our model with those of Burdett and Judd (1983). The first difference is that the Burdett and Judd model does not allow unemployment (every consumer buys exactly one unit), whereas in our model some workers remain unemployed. The source of this difference is the assumption in Burdett and Judd that each worker gets at least one job offer, and since in their equilibrium all firms make offers above the reservation wage, there can be no unemployment in their model. Any searching worker in our model may find that none of the firms he sampled are willing to hire him at any wage above the reservation wage (i.e., none of the firms he encounters are hiring), in which case he remains unemployed.

The second difference is that, while equilibria corresponding to type 1 and type 2 are present in the Burdett and Judd model, the hybrid equilibrium, type 3, cannot occur. The type 3 equilibrium in our model is driven by the concavity of the production function and by the assumption that not all workers are employed. Consider a version of our model with constant marginal costs as in Burdett and Judd, say $F(l) = l$. In this case, the wage rate equals the marginal and average cost of production. Therefore, given some entry costs, K, the wage rate must be smaller than the marginal product of labor, or firms would have earned negative profits. However, whenever the marginal productivity exceeds the wage rate, profits increase with the number of workers employed. Hence, there can be no mass of firms offering the same wage rate, since by raising its wage offer infinitesimally, a firm could attract more workers and accrue greater profits. In our model, the marginal product of labor declines. At w^*, where the wage rate equals the marginal product of labor, no firm has an incentive to hire more workers. Thus, declining marginal productivity of labor allows for a mass point at w^*, yielding our type 3 equilibrium.

The labor market in our model is characterized by two functions, $G(w)$ and $e(w)$. Proposition 1 reduces the problem of finding two *functions* to a problem of finding four *numbers*; $\underline{w}$, $\overline{w}$, ρ, and x. In proposition 2, we characterize the relationship between these variables if and only if the zero-profit constraint is maintained (proofs of propositions 2–4 are in the appendix).

PROPOSITION 2. In a steady state equilibrium $\underline{w}$, $\overline{w}$, ρ, and x satisfy the following conditions.

i) If $\rho = 0$, then

$$\underline{w} = \frac{F[x(1-\theta)^{n-1}] - K}{x(1-\theta)^{n-1}}$$

$$x \le l^*$$

$$\overline{w} = \frac{F(x) - K}{x}.$$

ii) If $0 < \rho \le \theta$, then $\underline{w}$ is as in (i)

$$x = \frac{n\rho l^*}{1-(1-\rho)^n}.$$

$$\overline{w} = \frac{F[x(1-\rho)^{n-1}] - K}{x(1-\rho)^{n-1}}.$$

iii) If $\rho = \theta$, then $\overline{w}$ does not exist, since all the firms offer w^*

$$\underline{w} \ge \frac{F[x(1-\theta)^{n-1}] - K}{x(1-\theta)^{n-1}}$$

$$x = \frac{n\theta l^*}{1 - (1 - \theta)^n}.$$

To prove propositions 1 and 2, we invoke firms' optimization and the zero-profit constraint. Proposition 3 derives the worker's optimal job acceptance rule.

PROPOSITION 3. In a steady state equilibrium,

i) If $\theta > \rho \geq 0$, then

$$\underline{w} = \beta(1 - \theta)\left[w^* - (w^* - \overline{w})(1 - \rho)^n - \int_{\underline{w}}^{\overline{w}} G(w)^n dw\right].$$

ii) If $\rho = \theta$, then

$$\underline{w} = w^*\left[1 - \frac{1 - \beta(1 - \theta)}{1 - \beta(1 - \theta)^{n+1}}\right].$$

Propositions 1, 2, and 3 provide the necessary and sufficient conditions that characterize the steady state equilibrium. In proposition 4 we establish that the assumptions made thus far are sufficient for its existence.

PROPOSITION 4. If F is strictly increasing, concave, twice continuously differential, and there exists $l < \infty$ such that $F(l) > K$, then there exists an equilibrium.

Uniqueness proves more difficult to show analytically. To establish whether the equilibrium is unique, and to evaluate the values of n associated with various equilibrium types, we assume a specific production function.

Specific Production Function

We derived analytically three necessary and sufficient conditions for the existence of each equilibrium type for a particular form of the production function

$$F(l) = zl^\alpha.$$

The above conditions allow us to determine the values of parameters for which each equilibrium type exists.[10] Since each of the three conditions is necessary for the existence of a respective equilibrium type, uniqueness is assured if and only if these conditions are mutually exclusive. We simulated these conditions for a tight grid of parameter values: $n \in (2, 100)$, $\alpha \in (0.05, 0.95)$, $\beta \in (0.8, 0.999)$, and $\theta \in (0.01, 0.99)$, and found

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[10] The conditions are not presented here but are available on request.

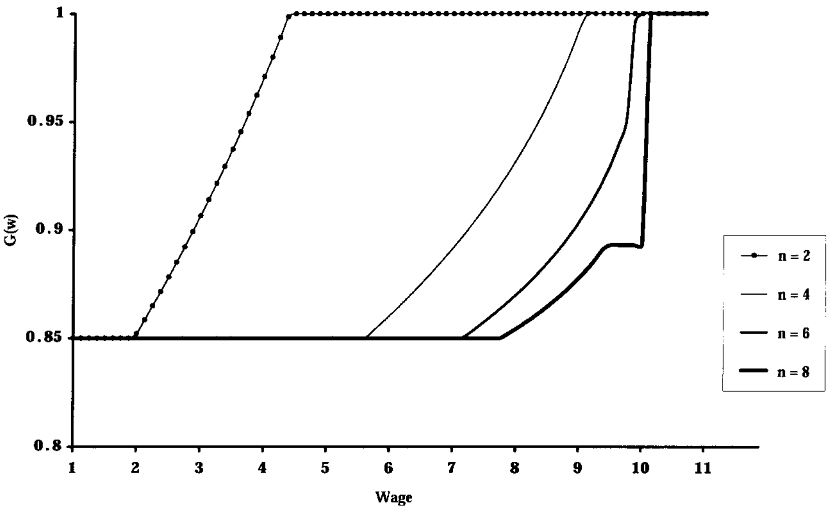

FIG. 1.—Wage distributions, $G(w)$, for varying n ($\theta = 0.15, \beta = 0.99, \alpha = 0.5, w^* = 10$).

them to be mutually exclusive everywhere.[11] The results can be summarized as follows: there exist $\underline{n}$ and $\overline{n}$, which are functions only of α, θ, and β, such that $\underline{n} < \overline{n}$, and

type 1 equilibrium exists if and only if (iff) $1 < n \leq \underline{n}$;
type 2 equilibrium exists iff $\overline{n} \leq n$;
type 3 equilibrium exists iff $\underline{n} < n < \overline{n}$.

To illustrate the evolution of the equilibrium, we present an example with $\alpha = 0.5$ and $\beta = 0.99$. Figure 1 presents the steady state equilibrium wage distributions, $G(w)$, for $\theta = 0.15$, for several values of n. At $n = 2$, the distribution of wages is approximately uniform between two and four, sharply below the competitive wage, $w^* = 10$. As n increases from two to six, the support shifts to 7–10, and the distribution is skewed toward the higher wage rates. The qualitative change occurs when $n = 8$: 11 out of every 15 firms offer the competitive wage, and only the remaining four offer wages between eight and 9.5. To get a better sense of the actual value of the threshold parameters at which the economy shifts from one equilibrium type to another, we plot in figure 2 $\underline{n}$ and $\overline{n}$ as functions of θ. The two curves divide the plane of θ and n into three regions, corresponding to three equilibrium types. For example, when $\theta = 0.15$, then whenever n is smaller than six, the economy resembles a

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[11] It turns out that K does not affect the equilibrium type. This is not a feature associated with a particular production function, but it is a more general phenomenon in our model. We discuss the implications of this feature later.

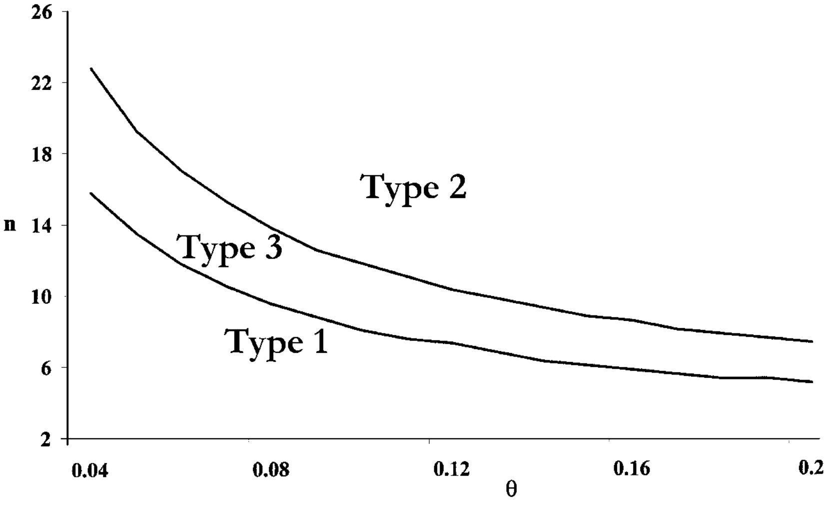

FIG. 2.—Equilibrium types associated with values of n and θ ($\alpha = 0.5$, $\beta = 0.99$)

search economy. If n equals seven or eight, then there is a wide range of wage offers, some of which are the competitive wage. If n is greater than nine, then all the firms offer the competitive wage. However, these values of n are sensitive to changes in θ. Thus, in an economy (or industry) with a turnover rate of 0.4, $n = 4$ is sufficient to generate a perfectly competitive labor market. On the other extreme, if $\theta = 0.05$, the required level of n to generate competition is 20.

Figure 2 illustrates two phenomena. First, that given θ, whenever n is small, there is a type 1 (search) equilibrium. As n grows larger, the economy shifts from the search equilibrium to the hybrid equilibrium until at large enough n, to the competitive equilibrium. Second, the larger the turnover rate, θ, the smaller the level of n required to generate competition.

The intuition underlying the first phenomenon is straightforward. To attract a worker who has approached a firm, the firm has to offer him a wage exceeding all his other $n - 1$ offers. However, the more offers a searching worker receives, the higher his expected highest wage offer. Therefore, a higher n increases the competition among firms for workers, inducing them to offer higher wages in order to match the workers' outside options.

The second phenomenon is more intriguing. The intuition underlying this result is that a larger θ implies a higher turnover of firms, which increases the chances of unemployed workers encountering hiring firms. This, in turn, raises the workers' expected highest wage offer, inducing the firms to offer higher wages.

IV. Implications of the Model

In this section we derive the aggregate measure of employment/unemployment and characterize the relation to parameters of the model. Let f denote the measure of workers laid off every period. Recall that L denotes the aggregate employment level, and $U \equiv 1 - L$ denotes the pool of unemployed workers at the end of every period. Exiting firms lay off their workers, who join the ranks of the unemployed and immediately start searching; altogether $U + f$ workers search every period.

Each searching worker solicits offers from n randomly selected firms. As we postulated above, only the newly entering firms have openings. The probability that a worker sampling one firm at random does not receive an offer equals the probability that he encounters one of the old firms: $G(\underline{w}) = 1 - \theta$. Thus, the probability of finding at least one acceptable offer during n trials is $[1 - (1 - \theta)^n]$. In the steady state the measure of laid-off workers, f, has to equal the measure of the newly employed, that is,

$$f \equiv \theta L = (U + f)[1 - (1 - \theta)^n]. \tag{4}$$

Substituting $U \equiv 1 - L$, we obtain the equilibrium aggregate employment, L:

$$L = \frac{1 - (1 - \theta)^n}{1 - (1 - \theta)^{n+1}}. \tag{5}$$

Consequently, the pool of unemployed is

$$U \equiv 1 - L = \frac{\theta(1 - \theta)^n}{1 - (1 - \theta)^{n+1}}. \tag{6}$$

Figures 3 and 4 present the equilibrium unemployment level as functions of n and θ, respectively. While our goal is not to calibrate the model but, rather, to illustrate the equilibrium wage distribution for various n and θ, nevertheless, we obtain reasonable levels of unemployment.

We identify four interesting implications of our model. First, the model exhibits an interesting dichotomy: aggregate employment is determined by the proportion of exiting (and entering) firms, θ, and by the search technology, n, but not by the production technology (see eq. [5]). On the other hand, the competitive wage and the optimal firm size are determined exclusively by $F(l)$ and K, as in the Walrasian equilibrium. This holds even for low levels of search intensity, where the other features of the Walrasian equilibrium do not hold. Thus, changes in the rate of firm mortality and search technology affect employment, but not wages, while changes in the production technology affect wages, but not employment. Consequently, the comparative statics on the comovement of wages and

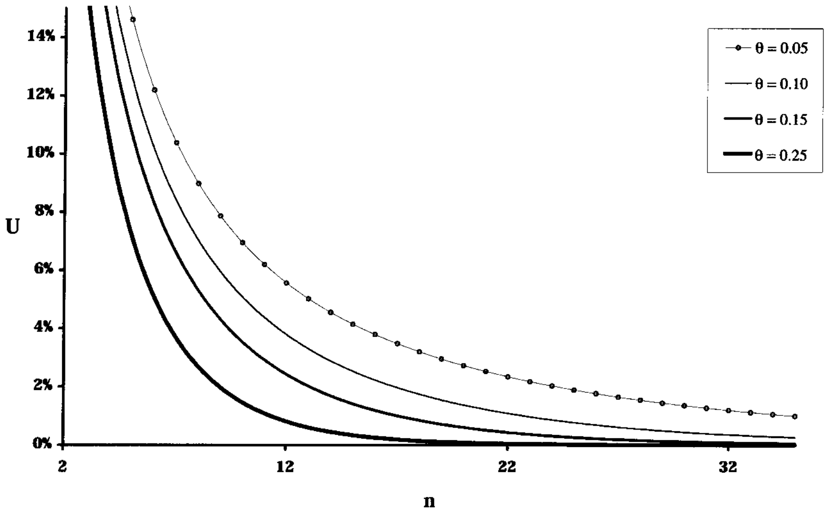

Fig. 3.—Equilibrium unemployment as a function of n

employment derived from our model are hard to replicate in either the Walrasian or the conventional search framework.

This dichotomy reflects the fact that the probability of finding a hiring firm depends solely on the proportion of new firms entering the economy, θ, and the number of draws, n. Since θ and n are taken to be exogenous, technological changes to F or to K affect wages but do not affect the aggregate employment.

Second, while the aggregate employment does not depend on the equilibrium type, total value added certainly does. Since every firm makes zero profits and K represents a real cost to the economy, the net value added equals the total wage bill. Since type 2 equilibrium yields the highest wage rate, w^*, and the lowest unemployment, it also yields the highest value added. This result differs from the conclusions in Acemoglu and Shimer (1997), who claim that a single wage equilibrium cannot be efficient. In their model, however, single wage equilibrium gives firms monopsonistic power by making workers' search futile. In our model even when all hiring firms offer the same wage, unemployed workers have an incentive to search, since they do not know which firms have vacancies.

Third, as n increases, the equilibrium is more likely to be of type 2, while at the same time the level of employment increases as well (see fig. 3). This implies that the net value added increases with n. As n gets large, the level of employment continuously converges to the full employment that characterizes the Walrasian model. This is consistent with the notion that n is a measure of information available to job seekers.

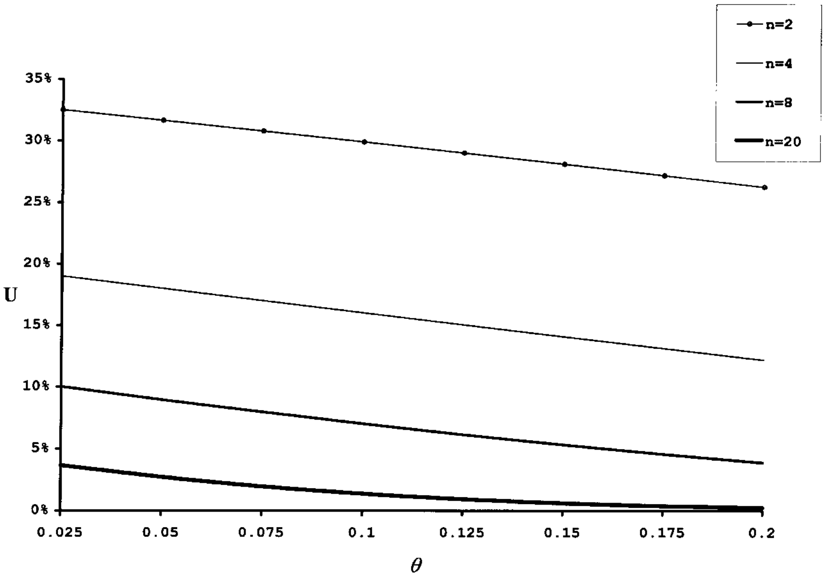

FIG. 4.—Equilibrium unemployment as a function of θ

As an example, let us consider the economy described in the previous section, where $F(l) = 200\sqrt{l}, K = 1{,}000, \theta = 0.15$, and $\beta = 0.99$. Compare this economy in the case where $n = 9$, and all the firms offer the competitive wage, to the case where $n = 2$. Recall that when $n = 9$, each firm employs $l^* = 100$, whereas if $n = 2$, firms pay wages between $\underline{w} = 1.9$ and $\overline{w} = 4.1$, and they employ between 27.7 and 32 workers. The firms in the latter case are between a quarter and a third of the efficient size. Moreover, aggregate unemployment in the former case is 5%, versus 28% in the latter (see eq. [5]). Thus, an increase in the number of searches from two to nine yields a dramatic reduction in unemployment and a large increase in output accompanied by a reduction in capital outlays, K.

Fourth, as θ increases, the effects are similar to that of an increase in n: the equilibrium is more likely to be of type 2, and the level of employment increases (see fig. 4); the net value added also increases in θ. In addition, the aggregate level of employment is positively correlated with job turnover. In other words, higher steady state rates of jobs creation and destruction are associated with lower levels of unemployment. We can, therefore, suggest two empirical implications of this model: first, an economy with more labor turnover is likely to have a lower steady state level of frictional unemployment. Second, since the probability of finding a job per period increases in θ, the average length of unemployment spells decline in θ as well.

We performed an exploratory analysis of 1992–93 unemployment data for the 20 largest Organization for Economic Cooperation and Development (OECD) countries (see OECD 1996). We use the proportion of newly unemployed (less than a month) out of the age 15–64 population as a proxy for the turnover rate in the economy, θ. We calculate the correlation between the turnover and the percentage of long-term unemployed (for more than 6 months); the model predicts that this correlation should be negative.

The estimated correlation coefficient between the proportion of newly unemployed and long-term unemployed in our sample is -0.48 in 1992 and -0.25 in 1993 (both are significant). Note that the comparative statics in our model are between steady states, whereas some countries in the sample experienced severe employment shocks during these years. To mitigate this problem we restrict our attention to 10 countries with the lowest year-to-year percentage changes in unemployment. In this sample, the correlation coefficients become -0.73 in 1992 and -0.59 in 1993. These results are consistent with our model; however, more rigorous empirical tests are required. These are left for future research.

An interesting question is whether these effects are robust to perturbations in our model. To assess this, we sketch an extension of our model in which firms are allowed to increase their labor force gradually over time, albeit at the same wage rate. In that case, firms will hire additional workers until they either suffer a bad shock and exit or reach the point where the marginal product of labor equals their wage offer. In the latter case, the firms put up "no vacancy" signs and reject all their applicants. Formal analysis of this model is hard, but we may venture to compare some of its implications to our model.

The perfect dichotomy that characterizes our model is no longer valid in this version, since the proportion of hiring firms depends on the distribution of wages. Firms that offer higher wages will fill their vacancies and stop hiring faster. It can be shown, for example, that in equilibrium, firms that offer the competitive wage will fill all their vacancies in the first period after they enter and will not hire thereafter. But since w^* and l^* are the same in both versions of the model, type 2 (the competitive) equilibria will have exactly the same properties in both versions, including the dichotomy of wage and unemployment determination. Moreover, since there are firms that offer w^* in type 3 equilibria, these implication are likely to survive to some extent, at least for high enough ρ. To conclude, although the dichotomy and other implications derived in this article depend on our simplifying assumptions, we believe that our qualitative results are likely to survive more realistic specifications of our model.

V. Summary

We present a search model that deviates from the traditional search literature by allowing each worker to sample n firms simultaneously. Exogenously given, n serves as a natural proxy for labor market information available to workers; the larger the number of firms sampled during a period, the more information a worker has about which firms are hiring and the wages offered by particular firms. We find that for low levels of search intensity (small n) the equilibrium is characterized by wage dispersion below the marginal product of labor, as in typical search models. When n increases, some firms start offering the competitive wage, while the rest continue to offer lower wages. Finally, when the number of searches per period becomes sufficiently large, all the firms offer the competitive wage. Surprisingly, the latter equilibrium type can obtain at quite low levels of search intensity. While not presented, we can also show that these results can be obtained in a model where workers face a well-behaved search cost function and choose n optimally.

Aggregate employment in this model is determined by a firm's mortality and by the search intensity of workers. Wages are determined solely by technology. This feature of the model leads to empirical predictions that are consistent with stylized facts but are hard to replicate using the Walrasian setup. More specifically, changes in the variability of demand shocks can reduce employment without affecting wages, whereas technological changes can affect wages without altering the level of employment. Thus, our economy generates comovements of wages and employment that could be interpreted as reflecting wage rigidities.

Appendix

Appendix Proofs

Proof of proposition 2. By proposition 1,

$$e(w^*) = \frac{1 - (1 - \rho)^n}{\rho n},$$

and every firm makes offers to all applicants, that is, $l^* = xe(w^*)$. Hence,

$$x = \frac{n\rho l^*}{1 - (1 - \rho)^n} \ \forall \rho > 0.$$

When $\rho = 0$, then by proposition 1, $e(\overline{w}) = G(\overline{w})^{n-1} = 1$. This and the zero-profit constraint imply that $l(\overline{w}) = xe(\overline{w})$. Since $l(\overline{w}) \leq l^*$, it follows that $x \leq l^*$.

From proposition 1,

$$G(\overline{w}) = \left[\frac{l(\overline{w})}{x}\right]^{\frac{1}{n-1}} = 1 - \rho,$$

thus

$$l(\overline{w}) = x(1 - \rho)^{n-1}.$$

Substituting $l(\overline{w})$ into the zero-profit constraint, $F[l(\overline{w})] - \overline{w}l(\overline{w}) - K = 0$, and rearranging terms yields

$$\overline{w} = \frac{F[x(1-\rho)^{n-1}] - K}{x(1-\rho)^{n-1}}.$$

Similarly,

$$l(\underline{w}) = x(1 - \theta)^{n-1},$$

thus

$$\underline{w} = \frac{F[x(1-\theta)^{n-1}] - K}{x(1-\theta)^{n-1}}.$$

This concludes the proof of proposition 2.

Proof of proposition 3. Let $V(w)$ denote the value function (expected lifetime discounted earnings) of a worker whose highest wage offer in this period is w and who behaves optimally. Let V denote the expected value of lifetime earnings prior to search. Then

$$V(w) = \max\{\beta V, w + \theta\beta V + (1-\theta)\beta V(w)\},$$

where βV is the value of rejecting the offer and searching again in the next period, and $w + \theta\beta V + (1-\theta)\beta V(w)$ is the value of accepting w. The reservation wage $\underline{w}$ makes the worker indifferent between the two options:

$$\beta V = \underline{w} + \theta\beta V + (1-\theta)\beta V(\underline{w}) = V(\underline{w}).$$

Hence,

$$V = \frac{\underline{w}}{\beta(1-\beta)(1-\theta)}, \tag{A1}$$

and for $w \geq \underline{w}$:

$$V(w) = \frac{w + \theta\beta V}{1 - (1-\theta)\beta}.$$

On the other hand, V is the expected value of the highest wage obtained in n searches. Since by claim 3 all the firms issue offers to all their applicants, the highest offer out of n independent wage draws is distributed according to a cumulative distribution function $G(w)^n$, and

$$V = \beta G(\underline{w})^n V + \int_{\underline{w}}^{\infty} V(w) dG(w)^n = \beta(1-\theta)^n V + \int_{\underline{w}}^{w^*} V(w) dG(w)^n$$

$$= \beta(1-\theta)^n V + \frac{(1-(1-\theta)^n)\theta\beta V}{1-(1-\theta)\beta} + \int_{\underline{w}}^{w^*} \frac{w}{1-(1-\theta)\beta} dG(w)^n,$$

or

$$V = \frac{\int_{\underline{w}}^{w^*} w dG(w)^n}{(1-\beta)[1-\beta(1-\theta)^{n+1}]}.$$

Substituting V from equation (A1) yields

$$\underline{w} = \frac{\beta(1-\theta)}{1-\beta(1-\theta)^{n+1}} \int_{\underline{w}}^{w^*} w dG(w)^n. \tag{A2}$$

Integrating by parts and substituting the values of $G(\underline{w})$ and $G(\overline{w})$ from proposition 1, we obtain

$$\begin{aligned} \int_{\underline{w}}^{w^*} w dG(w)^n &= \int_{\underline{w}}^{\overline{w}} w dG(w)^n + w^*[1-(1-\rho)^n] \\ &= \overline{w}(1-\rho)^n - \underline{w}(1-\theta)^n \\ &\quad - \int_{\underline{w}}^{\overline{w}} G(w)^n dw + w^*[1-(1-\rho)^n]. \end{aligned}$$

Equation (A2) can be rewritten as

$$\underline{w} = \begin{cases} \beta(1-\theta)\left[w^* - (w^* - \overline{w})(1-\rho)^n - \int_{\underline{w}}^{\overline{w}} G(w)^n dw\right] & \text{if } \theta > \rho \geq 0 \\ w^*\left[1 - \dfrac{1-\beta(1-\theta)}{1-\beta(1-\theta)^{n+1}}\right] & \text{if } \rho = \theta \end{cases}.$$

This concludes the proof of proposition 3.

Proof of proposition 4. Compare the following two terms:

$$\frac{F\left[\frac{n\theta l^*}{1-(1-\theta)^n}(1-\theta)^{n-1}\right] - K}{\frac{n\theta l^*}{1-(1-\theta)^n}(1-\theta)^{n-1}}$$

and

$$w^{*}\left[1-\frac{1-\beta(1-\theta)}{1-\beta(1-\theta)^{n+1}}\right].$$

There are two possibilities:

CASE 1.

$$\frac{F\left[\frac{n\theta l^{*}}{1-(1-\theta)^{n}}(1-\theta)^{n-1}\right]-K}{\frac{n\theta l^{*}}{1-(1-\theta)^{n}}(1-\theta)^{n-1}}\leq w^{*}\left[1-\frac{1-\beta(1-\theta)}{1-\beta(1-\theta)^{n+1}}\right].$$

CASE 2:

$$\frac{F\left[\frac{n\theta l^{*}}{1-(1-\theta)^{n}}(1-\theta)^{n-1}\right]-K}{\frac{n\theta l^{*}}{1-(1-\theta)^{n}}(1-\theta)^{n-1}}> w^{*}\left[1-\frac{1-\beta(1-\theta)}{1-\beta(1-\theta)^{n+1}}\right].$$

We analyze the two cases separately.

CASE 1. In this case, $\rho = \theta$ and

$$x^{*}=\frac{n\theta l^{*}}{1-(1-\theta)^{n}}$$

constitute a type 2 equilibrium as

$$\frac{F\left[x^{*}(1-\theta)^{n-1}\right]-K}{x^{*}(1-\theta)^{n-1}}=\frac{F\left[\frac{n\theta l^{*}}{1-(1-\theta)^{n}}(1-\theta)^{n-1}\right]-K}{\frac{n\theta l^{*}}{1-(1-\theta)^{n}}(1-\theta)^{n-1}}$$

$$\leq w^{*}\left[1-\frac{1-\beta(1-\theta)}{1-\beta(1-\theta)^{n+1}}\right]=\underline{w},$$

where the inequality stems from the condition characterizing case 1 and the last equality is satisfied in a type 2 equilibrium (see sec. ii in proposition 3). Thus, section iii in proposition 2 and section ii in proposition 3 are satisfied, which proves that in case 1 there is a type 2 equilibrium with $\rho = \theta$ and

$$x^{*}=\frac{n\theta l^{*}}{1-(1-\theta)^{n}}.$$

CASE 2. Let us define x^{*} as before:

$$x^{*}\equiv\frac{n\theta l^{*}}{1-(1-\theta)^{n}},$$

and $\underline{x}$ that satisfies

$$\frac{F[\underline{x}(1-\theta)^{n-1}]-K}{\underline{x}(1-\theta)^{n-1}}=0.$$

Since

$$\frac{F[x(1-\theta)^{n-1}]-K}{x(1-\theta)^{n-1}}$$

is continuously increasing with x, unbounded below as x approaches zero and by assumption $F[x(1-\theta)^{n-1}]-K>0$ for large enough x, it follows that such $\underline{x}$ exists and is unique. By the condition characterizing case 2,

$$\begin{aligned}\frac{F\left[x^*(1-\theta)^{n-1}\right]-K}{x^*(1-\theta)^{n-1}} &= \frac{F\left[\frac{n\theta l^*}{1-(1-\theta)^n}(1-\theta)^{n-1}\right]-K}{\frac{n\theta l^*}{1-(1-\theta)^n}(1-\theta)^{n-1}}\\ &> w^*\left[1-\frac{1-\beta(1-\theta)}{1-\beta(1-\theta)^{n+1}}\right]\\ &= w^*\left[\frac{\beta(1-\theta)[1-\beta(1-\theta)^n]}{1-\beta(1-\theta)^{n+1}}\right]>0.\end{aligned}$$

Thus,

$$\frac{F[\bar{x}(1-\theta)^{n-1}]-K}{\bar{x}(1-\theta)^{n-1}}>\frac{F[\underline{x}(1-\theta)^{n-1}]-K}{\underline{x}(1-\theta)^{n-1}},$$

and it follows that $\underline{x}<x^*$.

Next, we define a function $T(x)$ as follows:

$$\begin{aligned}T(x) &= \frac{F[x(1-\theta)^{n-1}]-K}{x(1-\theta)^{n-1}}\\ &\quad -\beta(1-\theta)\left[w^*-(w^*-\overline{w})(1-\rho)^n-\int_{\underline{w}}^{\overline{w}}G(w)^n dw\right],\end{aligned}$$

where

$$\underline{w}=\frac{F[x(1-\theta)^{n-1}]-K}{x(1-\theta)^{n-1}},$$

$$\overline{w}=\frac{F[x(1-\rho)^{n-1}]-K}{x(1-\rho)^{n-1}},$$

and if $x \leq l^*$, then $\rho = 0$, and if $x > l^*$, then ρ solves

$$x = \frac{n\rho l^*}{1 - (1 - \rho)^n}.$$

Since

$$\lim_{\rho \to 0} \frac{n\rho l^*}{1 - (1 - \rho)^n} = l^*,$$

it follows that for $x > 0$, $T(x)$ is continuous. By the definition of $\underline{x}$,

$$\begin{aligned} T(\underline{x}) &= -\beta(1-\theta)\left[w^* - (w^* - \overline{w})(1-\rho)^n - \int_0^{\overline{w}} G(w)^n dw\right] \\ &= -\beta(1-\theta)\int_0^{w^*} w dG(w)^n < 0, \end{aligned}$$

where the last equality is obtained by integrating by parts $\int_{\underline{w}}^{w^*} w dG(w)^n$ and substituting $G(\underline{w}) = (1-\theta)^n$ and $G(\overline{w}) = (1-\rho)^n$.

From the proof of proposition 3, we know that

$$w^*\left[1 - \frac{1-\beta(1-\theta)}{1-\beta(1-\theta)^{n+1}}\right] \geq \beta(1-\theta)\left[w^* - (w^* - \overline{w})(1-\rho)^n - \int_{\underline{w}}^{\overline{w}} G(w)^n dw\right].$$

Thus, by the definition of $\overline{x}$,

$$\begin{aligned} T(\bar{x}) &= \frac{F\left[\frac{n\theta l^*}{1-(1-\theta)^n}(1-\theta)^{n-1}\right] - K}{\frac{n\theta l^*}{1-(1-\theta)^n}(1-\theta)^{n-1}} \\ &\quad - \beta(1-\theta)\left[w^* - (w^* - \overline{w})(1-\rho)^n - \int_{\underline{w}}^{\overline{w}} G(w)^n dw\right] \\ &\geq \frac{F\left[\frac{n\theta l^*}{1-(1-\theta)^n}(1-\theta)^{n-1}\right] - K}{\frac{n\theta l^*}{1-(1-\theta)^n}(1-\theta)^{n-1}} - w^*\left[1 - \frac{1-\beta(1-\theta)}{1-\beta(1-\theta)^{n+1}}\right] > 0, \end{aligned}$$

where the last inequality states the condition characterizing case 2.

We have shown that $T(x)$ is a continuous function on $[\underline{x}, x^*]$ with

$T(\underline{x}) < 0$ and $T(x^*) > 0$. Therefore, by the Mean Value Theorem, there exists $\underline{x} \leq x_0 \leq \bar{x}$ for which $T(x_0) = 0$. Substituting $x = x_0$, we obtain

$$\underline{w} = \frac{F[x_0(1-\theta)^{n-1}] - K}{x_0(1-\theta)^{n-1}}$$

and

$$\overline{w} = \frac{F[x_0(1-\rho)^{n-1}] - K}{x_0(1-\rho)^{n-1}}.$$

By propositions 2 and 3, x_0, $\underline{w}$, and $\overline{w}$ constitute an equilibrium of either type 1, if $x_0 \leq l^*$ and $\rho = 0$, or type 3 if $x_0 > l^*$ and ρ, which solves

$$x_0 = \frac{n\rho l^*}{1-(1-\rho)^n}.$$

This completes the proof of proposition 4.

References

Acemoglu, Daron, and Shimer, Robert. “Wage and Technology Dispersion.” *Review of Economic Studies* 67, no. 4 (2000): 585–608.

Albrecht, James W., and Axell, Bo. “An Equilibrium Model of Search Unemployment.” *Journal of Political Economy* 92 (October 1984): 824–40.

Albrecht, James, and Jovanovic, Boyan. “The Efficiency of Search under Competition and Monopsony.” *Journal of Political Economy* 94 (December 1986): 1246–57.

Burdett, Kenneth, and Judd, Kenneth, L. “Equilibrium Price Distributions.” *Econometrica* 51 (1983): 955–70.

Burdett, Kenneth, and Mortensen, Dale, T. “Equilibrium Wage Differentials and Employer Size.” Discussion Paper no. 860. Evanston, IL: Northwestern University, Center for Mathematical Studies in Economics and Management Science, 1989.

Burdett, Kenneth, and Vishwanath, Tara. “Balanced Matching and Labor Market Equilibrium.” *Journal of Political Economy* 96 (1988): 1048–65.

Butters, Gerard. “Equilibrium Distribution of Sales and Advertising Prices.” *Review of Economic Studies* 49, no. 2 (1977): 217–27.

Chalkley, Martin. “Monopsony Wage Determination and Multiple Equilibria in a Non-linear Search Model.” *Review of Economic Studies* 58, no. 1 (1991): 181–93.

Diamond, Peter A. “A Mode of Price Adjustment.” *Journal of Economic Theory* 3 (1971): 156–68.

———. “Wage Determination and Efficiency in Search Equilibrium.” *Review of Economic Studies* 49 (April 1982): 217–27.

Lang, Kevin. “Persistent Wage Dispersion and Involuntary Unemployment.” *Quarterly Journal of Economics* 106 (1991): 181–202.

Montgomery, James D. “Equilibrium Wage Dispersion and Interindustry

Wage Differentials." *Quarterly Journal of Economics* 106 (February 1991): 163–79.
OECD Job Study. *Unemployment in the OECD Area, 1950–1995*. Paris: OECD, 1996.
Pissarides, Christopher A. *Equilibrium Unemployment Theory*. 2d ed. Cambridge, MA: MIT Press, 2000.
Stigler, George. "The Economics of Information." *Journal of Political Economy*, 69, no. 3 (1961): 213–25.
Vroman, S. B. "No-Help-Wanted Signs and the Duration of Job Search." *Economic Journal* 95 (1985): 767–63.
Wilde, Louis. "Labor Market Equilibrium under Non-sequential Search." *Journal of Economic Theory* 16 (1977): 373–93.