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Pricing Information Goods in the Presence of Copying

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Abstract

The effects of (private, small-scale) copying on the pricing behavior of producers of information goods are studied within a unified model à la Mussa-Rosen (1978). When the copying technology involves a marginal cost and no fixed cost, producers act independently. In this simple framework, we highlight the trade-off between *ex ante* and *ex post* efficiency considerations (how to provide the right incentives to create whilst limiting monopoly distortions?). When the copying technology involves a fixed cost and no marginal cost, pricing decisions are interdependent. We investigate the strategic pricing game by focussing on some significant symmetric Nash equilibria.

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1 Introduction

Information can be defined very broadly as anything that can be digitized (i.e., encoded as a stream of bits), such as text, images, voice, data, audio and video (see Varian, 1998). Information is exchanged under a wide range of formats or packages (which are not necessarily digital). These formats are generically called *information goods*. Books, movies, music, magazines, databases, telephone conversations, stock quotes, web pages, news, etc. all fall into this category.

Most information goods are expensive to produce but cheap to reproduce. This combination of high fixed costs and low (often negligible) marginal costs implies that information goods are inherently *nonrival*.[1] Moreover, because reproduction costs are also potentially very low *for anybody other* than the creator of the good, information goods might be *nonexcludable*, in the sense that one person cannot exclude another person from consuming the good in question.

The degree of excludability of an information good (and hence the creator's ability to appropriate the revenues from the production of the good) can be enhanced by legal authority (typically by the adoption of laws protecting intellectual property) or by technical means (e.g., cable broadcast are encrypted, so-called "unrippable" CDs have recently appeared). However, complete excludability seems hard to achieve: simply specifying intellectual property laws does not ensure that they will be enforced; similarly, technical protective measures are often imperfect and can be "cracked". As a result, *illicit copying* (or piracy) cannot be completely avoided.

Over the last decade, the fast penetration of the Internet and the increased digitization of information have turned piracy of information goods (in particular music, movies and software) into a topic of intense debate. A selection of news headlines gathered recently (February-March 2002) illustrates the current extent of the debate. These headlines are about (i) a proposed anti-piracy bill in the US that would ultimately require computer and consumer electronics companies to build piracy-prevention software into their products, (ii) a man facing jail in California for Web sales of CDs, (iii) the release of new peer-to-peer file-sharing softwares aiming to replace Napster, (iv) music distributors estimating that retail sales may be down as much as 10 percent during the past year as consumers shift to new technologies like copying CDs and downloading songs, (v) music companies settling a lawsuit with a CD consumer who alleged that the CD she purchased did not meet consumer expectations because it could not be played on a computer, or (vi) a Taiwanese Web site that offers access to

[1] Nonrivalness in consumption is usually defined by saying that the consumption possibilities of one individual do not depend on the quantities consumed by others. This is equivalent to say that, for any given level of production, the marginal cost of providing the good to an additonal consumer is zero.

a huge library of films for just $1 each (and which, understandably, has drawn Hollywood's ire).[2]

Not surprisingly, economists have recently shown a renewed interest in information goods piracy. Here follows a selection of recent working papers which investigate a number of topical issues. Gayer and Shy (2001a) show the inefficiency of using hardware taxation to compensate copyright owners for infringements of their intellectual property (IP). In another article (Gayer and Shy, 2001b), the same authors investigate how producers of digital information goods can utilize the Internet's distribution channels, such as peer-to-peer systems, to enhance sales of their goods sold in store. The welfare implications of peer-to-peer distribution technologies are also the concern of Duchêne and Waelbroeck (2001); they show that the losses generated by illegal copies can be offset by the introduction of new products, which creates a positive surplus for their creators, as well as consumers. The idea that copyright infringement could be strategically promoted by creators is also explored by Ben-Shahar and Jacob (2001); they show, in a dynamic model, that creators might favor selective copyright enforcement as a form of predatory pricing in order to raise barriers to entry. Turning to policy matters, Harbaugh and Khemka (2001) argue that copyright enforcement targeted at high-value buyers raises copyright holder profits but, at the same time, increases piracy relative to no enforcement; therefore, they contend that either no enforcement or relatively extensive enforcement is the best policy against Internet piracy. In the same vein, Chen and Png (2001) examine how the government should set the fine for copying, tax on copying medium, and subsidy on legitimate purchases, while a monopoly publisher sets price and spending on detection; they conclude that government policies focussing on penalties alone would miss the social welfare optimum. Yoon (2001) also aims at determining the optimal level of copyright protection for an individual producer and for society as a whole. Finally, Hui, Png and Cui (2001) provide one of the rare attempts to estimate empirically the actual impact of piracy on the legitimate demand for information goods. Using international panel data for music CDs and cassettes, they find that the demand for both goods decreased with piracy.

These recent contributions revive the literature on the economics of copying and copyright, which was initiated some twenty years ago.[3] The seminal papers discussed the effects of photocopying and examined, among other things,

[2]Sources: (i) *Proposed anti-piracy bill draws fire*, by Stefanie Olsen (CNET News.com, March 25, 2002), (ii) *Man faces jail for Web sales of CDs*, by Lisa M. Bowman (CNET News.com, March 22, 2002), (iii) *Goodbye Napster, Hello Morpheus (and Audiogalaxy and Kazaa and Grokster...)*, by Erick Schonfeld (Business2.com, March 15, 2002), (iv) *Digital Music Fight Traps Retailers*, by Benny Evangelista (Newsfactor.com, March 12, 2002), (v) *Consumer claims victory in CD lawsuit*, by Lisa M. Bowman (CNET News.com, February 22, 2002), (vi) *Plug pulled on site selling $1 movies,* by John Borland (CNET News.com, February 19, 2002).

[3]With the notable exception of Plant (1934).

how publishers can appropriate indirectly some revenues from illegitimate users (Novos and Waldman, 1984, Liebowitz, 1985, Johnson, 1985, and Besen and Kirby, 1989). The economics of IP protection was then addressed more generally by Landes and Posner (1989) and Besen and Raskind (1991). Both papers discuss the following trade-off between *ex ante* and *ex post* efficiency considerations. From an *ex ante* point of view, IP protection preserves the incentive to create information goods, which (as argued above) are inherently public (absent appropriate protection, creators might not be able to recoup their potentially high initial creation costs). On the other hand, IP rights encompass various potential inefficiencies from an *ex post* point of view (protection grants de facto monopoly rights, which generates the standard deadweight losses; also, by inhibiting imitation, IP rights might limit the creators' ability to borrow from, or build upon, earlier works, and thereby increase the cost of producing new ideas). A third wave of papers paid closer attention to software markets and introduced network effects in the analysis. Conner and Rumelt (1991), Takeyama (1994), and Shy and Thisse (1999) share the following argument: because piracy enlarges the installed base of users, it generates network effects that increase the legitimate users' willingness to pay for the software and, thereby, potentially raises the producer's profits. Finally, and more closely related to this paper, Watt (2000) has surveyed–and extensively supplemented–the literature on the economics of copyright.

The aim of the present paper is to address several of the themes studied so far in the literature within a simple and unified model. Like a number of recent papers, we use the framework proposed by Mussa and Rosen (1978) for modelling vertical (quality) differentiation: copies are seen as lower-quality alternatives to originals (i.e., if copies and originals were priced the same, all consumers would prefer originals). In a benchmark model, we consider the market for a single information good. A monopolist must set the price for the original good, taking into account that consumers can alternatively acquire a lower-quality copy at a constant cost. The optimal strategy for the monopolist can usefully be described by using Bain (1956)'s taxonomy of an incumbent's behavior in the face of an entry threat. Unless the quality/price ratio of copies is very low (meaning that copying exerts no threat and will therefore be 'blockaded'), the producer will have to modify his behavior and decide whether to set a price low enough to 'deter' copying, or to 'accommodate' copying and make up for it by extracting a higher margin from fewer consumers of originals. Whatever the producer's optimal decision, we show that copying reduces the producer's profits but increases consumers surplus more than proportionally: as a result, copying (which amounts here to the provision of a cheaper and lower-quality alternative to a monopolized good) enhances social welfare.

The previous conclusion simply restates the *ex post* efficiency consideration of the traditional economic analysis of copying: if the information good was (legally or technically) better protected, the producer would fully enjoy his monopoly position and social welfare would be reduced. As argued above, such *ex post* inefficiency has to be balanced against *ex ante* considerations relating to creation costs. To incorporate this dimension, we extend the benchmark model by considering an arbitrary number of information goods. The Mussa-Rosen framework continues to apply for each information good. Moreover, to focus on the effects of copying, we assume that copying is the only source of interdependence between the demands for the various information goods. In particular, the goods are completely differentiated and consumers are assumed to have a sufficient (exogenous) budget to buy them all if they so wish.

Whether demands are interdependent or not depends on the nature of copying technology. In the spirit of Johnson (1985), we examine two extreme scenarios: the copying technology involves either a constant unit cost and no fixed cost, or a positive fixed cost and no marginal cost. In the former case, demands for originals are completely independent of one another: all producers act thus like the single-good monopolist of the benchmark model. Assuming a fixed creation cost that varies through producers, we can derive the number of information goods that are created at the long-run, free-entry, equilibrium. Obviously, copying reduces this number. We can then balance *ex ante* and *ex post* efficiency considerations and show that copying is likely to damage welfare in the long run (unless copies are a poor alternative to originals and/or are expensive to acquire).

The picture changes dramatically when the copying technology involves only a positive fixed cost. The demands for originals now become interdependent because consumers base their decision to invest in the copying technology on the cost of this technology and on the prices of *all* originals. Therefore, copying introduces strategic interaction between the producers of originals whom everything else otherwise separates. This strategic interaction makes the producers' pricing behavior (which takes the form of a simultaneous Bertrand game) more interesting–but also much more intricate–to analyze. Due to the complexity of the system of demands, we are unable to provide a complete characterization of the set of Bertrand-Nash equilibria. We shed, nevertheless, some light on symmetric equilibria in which copying is either blockaded, deterred or accommodated. We show, in particular, that the latter two equilibria rely on a set of rather restrictive conditions, as the incentives for unilateral deviation are high: producers tend to free-ride (by setting higher prices) when it comes to deterring copying, or they tend to undercut when it comes to accommodating copying.

The rest of the paper is organized as follows. In Section 2, we lay out a benchmark model with a single information good and we analyze the short-run welfare effects of copying. Then, we extend the benchmark model towards a multi-good setting in two different ways. In Section 3, we assume that copying involves a constant marginal cost and no fixed cost. Under this assumption, we examine the long-run welfare effects of copying. In Section 4, we assume instead that copying involves a positive fixed cost and no marginal cost. Due to the intricacies of the model under this alternative assumption, we leave welfare considerations aside and try instead to unravel the complex situation of strategic interaction that copying induces between producers of originals. We conclude and propose an agenda for future research in Section 5. Finally, we provide the proofs of the main propositions in Section 6.

2 A simple single-good model

We start by considering a very simple market for an information good supplied by a single producer.[4] We use the framework proposed by Mussa and Rosen (1978) for modelling vertical (quality) differentiation. There is a continuum of potential users who are characterized by their valuation, θ, for the information good. We assume that θ is uniformly distributed on the interval $[0, 1]$. Each user can obtain the information good in two different ways. One possibility is to *buy* the legitimate product (an "original") at price p. Originals are produced by a single producer at zero marginal cost. The alternative is to acquire a *copy* of the product at a cost $c \geq 0$. (In both cases, each user consumes at most one unit of the information good.) The two variants of the information good are indexed by their quality: let $s_o > 0$ denote the quality of an original and s_c (with $0 < s_c < s_o$), the quality of a copy.

The cost c can be thought of as the price of an illegitimate copy sold by some large-scale pirate, or as the cost of the copying medium. We discuss the precise nature of this cost at the end of the present section (for the moment, we refer to any means of using the information good without buying an original as 'copying'). The assumption that the quality of a copy is lower than the quality of an original ($s_c < s_o$) is common (see, e.g., Gayer and Shy, 2001a) and may be justified in several ways. In the case of analog reproduction, copies represent poor substitutes to originals. For instance, even the best photocopying loses information such as fine lines, fine print and true color images. Furthermore, copies of analog media are rather costly to distribute. Although this is no longer true for digital reproduction, originals might still provide users with a higher level of services, insofar as that they are bundled with valuable complementary products which can hardly be obtained otherwise.[5]

[4] Yoon (2001) independently developed a very similar model.

[5] For instance, many pieces of software come with free manuals and supporting services, or with discount on upgrades, all advantages that users who pirate the software will have to acquire at a positive price.

Accordingly, a user indexed by θ has a utility function defined by

$$U_\theta = \begin{cases} \theta s_o - p & \text{if buying an original,} \\ \theta s_c - c & \text{if copying,} \\ 0 & \text{if not using the information good.} \end{cases} \quad (1)$$

We assume that $c < s_c$, so that the user with the highest valuation for the product is better off copying than not using the product (otherwise, copying would trivially not be an issue).

2.1 User behavior

A user indexed by θ will buy the legitimate product under the following two conditions. First, buying must provide a higher utility than not using: $\theta s_o \geq p$. Second, buying must provide a higher utility than copying: $\theta s_o - p \geq \theta s_c - c$, which is equivalent to

$$\theta \geq \theta_1 \equiv \frac{p-c}{s_o - s_c}.$$

Clearly, the latter inequality cannot be met if originals are too expensive (if $p > s_o - s_c + c$, $\theta_1 > 1$ and no user buys an original).

On the other hand, the user θ will copy the product if the previous condition is reversed ($\theta < \theta_1$) and if copying provides a higher utility than not using: $\theta s_c - c \geq 0$, or

$$\theta \geq \theta_2 \equiv \frac{c}{s_c}.$$

These two inequalities are incompatible, meaning that no user finds it profitable to copy, if the price of originals is low enough. Indeed, $p \leq \bar{p} \equiv c s_o / s_c$ implies that $\theta_1 \leq \theta_2$. Note that the 'limit price' $\bar{p}$ decreases as copies become relatively more attractive (i.e., as c and s_o/s_c decrease).

There are thus three demand regimes. First, if the price of the legitimate product is too high (if $p \geq s_o - s_c + c$), then no user will buy the legitimate product.[6] Second, for intermediate prices (i.e., for $c s_o/s_c \leq p \leq s_o - s_c + c$), users indexed on $[\theta_1, 1]$ buy the legitimate product, users indexed on $[\theta_2, \theta_1]$ copy the product, others do not use. Finally, for low prices (i.e., for $0 \leq p \leq c s_o/s_c$), users indexed on $[p/s_o, 1]$ buy the legitimate product, whilst others do not use. Collecting the previous results, we can write the demand function for originals:

$$D(p) = \begin{cases} 0 & \text{for } p \geq s_o - s_c + c, \\ 1 - \frac{p-c}{s_o - s_c} & \text{for } \frac{c s_o}{s_c} \leq p \leq s_o - s_c + c, \\ 1 - \frac{p}{s_o} & \text{for } 0 \leq p \leq \frac{c s_o}{s_c}. \end{cases} \quad (2)$$

[6] Although users indexed on $[\theta_2, 1]$ are theoretically better off when they copy the product, copying does not appear as a feasible option when no originals circulate.

2.2 Producer's behavior

The producer's problem is to choose the price p of the legitimate product so as to maximize profits, $pD(p)$, with demand given by expression (2). The producer's problem is complicated by the fact that some users are better off copying the product once the price exceeds some threshold. There is thus a kink in the demand curve and the producer has to choose in which segment of the demand curve to operate. By analogy with Bain (1956)'s taxonomy of an incumbent's behavior in the face of an entry threat, we will say that the producer is either able to 'blockade' copying, or that he must decide whether to 'deter' copying or 'accommodate' it. Let us now define and compare these three options.

The producer blockades or deters copying. By setting a price sufficiently low, the producer can eliminate copying. The producer's maximization program is then

$$\max_p \pi(p) = p\left(1 - \frac{p}{s_o}\right) \text{ s.t. } p \leq \frac{cs_o}{s_c}. \tag{3}$$

The unconstrained profit-maximizing price and profits are easily computed as

$$p_b = \frac{s_o}{2},\ \pi_b = \frac{s_o}{4}.$$

This solution meets the constraints if and only if $c \geq s_c/2$. In this case, we can say that copying is actually *blockaded*: the producer safely sets his price as if copying was not a threat. Otherwise, copying cannot be blockaded but the producer modifies his behavior to successfully *deter* copying: he will choose the highest price compatible with the constraints, i.e.

$$p_d = \frac{cs_o}{s_c},\ \text{which implies } \pi_d = \frac{cs_o\,(s_c - c)}{s_c^2}.$$

The producer accommodates copying. The other option is to set a higher price and tolerate copying. The producer's program becomes

$$\max_p \pi(p) = p\left(1 - \frac{p-c}{s_o - s_c}\right) \text{ s.t. } \frac{cs_o}{s_c} \leq p \leq s_o - s_c + c. \tag{4}$$

Here, the unconstrained profit-maximizing price is equal to

$$p_a = \frac{s_o - s_c + c}{2},\ \text{which implies } \pi_a = \frac{(s_o - s_c + c)^2}{4\,(s_o - s_c)}.$$

This solution satisfies the constraints if and only if

$$\frac{s_o - s_c + c}{2} \geq \frac{cs_o}{s_c} \iff c \leq \frac{s_c\,(s_o - s_c)}{2s_o - s_c}.$$

If the latter condition is not met, it is easily checked that the corner solution is equivalent to copying deterrence.

Blockade, deter or accommodate? Collecting the previous results, we observe that the producer's optimal strategy depends on the relative attractiveness of copies (i.e., for a given value of s_o, on the values of c and s_c), as summarized in Proposition 1 and illustrated in Figure 1 (which is drawn for $s_o = 1$).

Proposition 1 *The producer's profit-maximization price is*

$$\left| \begin{array}{lll} p_b = \frac{s_o}{2}, & \textit{for } \frac{s_c}{2} \leq c \leq s_c & \text{(copying is blockaded)}, \\ \\ p_d = \frac{cs_o}{s_c}, & \textit{for } \frac{s_c(s_o - s_c)}{2s_o - s_c} \leq c \leq \frac{s_c}{2} & \text{(copying is deterred)}, \\ \\ p_a = \frac{s_o - s_c + c}{2}, & \textit{for } 0 \leq c \leq \frac{s_c(s_o - s_c)}{2s_o - s_c} & \text{(copying is accommodated)}. \end{array} \right.$$

[Insert Figure 1 about here]

2.3 Welfare effects of copying in the short run

Now that we have characterized the producer's pricing behavior, we are in a position to examine how copying affects welfare. Our *benchmark* is a hypothetical economy where copying would be infeasible; in this case, the producer would act as an unconstrained monopolist, which corresponds to the case of blockaded copying (defined above by the condition $c \geq s_c/2$). Consumer surplus (S_b) and social welfare (W_b) in this hypothetical economy are readily computed as follows:

$$\begin{aligned} S_b &= \int_{p_b/s_o}^{1} (\theta s_o - p) \, d\theta = s_o/8, \\ W_b &= \pi_b + S_b = 3s_o/8. \end{aligned}$$

Copies are relatively unattractive. If $s_c(s_o - s_c)/(2s_o - s_c) \leq c \leq s_c/2$, we know that the producer prefers to *deter* copying. In this case, the only effect of copying is to force the producer to set a lower price than the one he would set if copying exerted no threat ($p_d < p_b$). Although more users buy the legitimate product, the producer's profit falls, meaning that copying hurts him ($\pi_d < \pi_b$). However, the consumer surplus clearly increases and this increase offsets the reduction in profit, which results in an increase in social welfare (computed as the sum of consumer surplus and producer's profit): $W_d = s_o \left(s_c^2 - c^2\right)/2s_c^2 > W_b = 3s_o/8$. The possibility of making copies can be seen as a potential competition that disciplines the producer of the legitimate product in a welfare-enhancing way.

Copies are relatively attractive. For lower values of c (i.e., $c \leq s_c (s_o - s_c) / (2s_o - s_c)$), copying is *accommodated*. The welfare analysis becomes a bit more complicated and also more instructive. There are now users who get a positive surplus by copying the legitimate product and they have to be taken into account in the welfare analysis. Consider first the producer. Being just a threat (as in the previous case) or an actual fact (as here), copying has the same effect on the producer's pricing behavior: price has to go down (though less than under the deterrence option, $p_d < p_a < p_b$) and the increased demand this generates is not enough to prevent profit from falling ($\pi_a < \pi_b$). So, as in the previous case, the producer of the legitimate product suffers from copying. It can be argued that, from a social point of view, there is no reason to worry about the previous result: copying has the advantage of breaking down the monopoly the producer would enjoy otherwise. As we have just shown, copying leads to a lower price and a higher quantity consumed: legitimate users enjoy thus a larger surplus. Moreover, if we also incorporate the surplus enjoyed by illegal users, we find again that *copying has a positive impact on welfare*.

To establish this result, we compute the surplus for legitimate and illegitimate users when copying is accommodated respectively as

$$SL_a = \int_{\frac{p_a - c}{s_o - s_c}}^{1} (\theta s_o - p_a)\, d\theta \text{ and } SI_a = \int_{\frac{c}{s_c}}^{\frac{p_a - c}{s_o - s_c}} (\theta s_c - c)\, d\theta.$$

We then compute social welfare as $W_a = \pi_a + S_a$, with $S_a = SL_a + SI_a$, and we observe that

$$W_a - W_b = \frac{1}{8} \frac{(4s_o - s_c)\, c^2 + s_c\, (s_c - 2c)\, (s_o - s_c)}{s_c\, (s_o - s_c)} > 0. \qquad (5)$$

We record the above two findings in the following proposition.

Proposition 2 *So long as it cannot be blockaded, copying improves social welfare in the short run.*

The intuition underlying Proposition 2 is obvious. By introducing a cheaper imperfect substitute for originals, copying reduces the monopoly power of the producer and, thereby, increases social welfare. This result must, however, be qualified in one important way. Most generally, the creation of information goods involves substantive fixed 'first-copy' costs. So far, we have abstracted this fixed cost away by assuming implicitly that the producer could cover it even when he had to accommodate copying. That is, noting the fixed creation cost by F, we have assumed that $\pi_a > F$. It is only under that assumption that the above result holds. Indeed, if we had instead that $\pi_b > F > \pi_a$, the producer would not create the information good if he had no other choice than to accommodate copying. In such a case, copying would clearly reduce social welfare.

Long-run perspective. In the next two sections, we examine the previous issue more closely by considering a multi-product framework. More precisely, we extend the benchmark model by assuming that users now have the possibility to consume from a set G of information goods (with $|G| \geq 2$). As before, consumers choose, for each product, to either buy an original, make a copy, or not consume at all. We make the following assumptions about these three possibilities.

- *No use.* As before, the utility from not consuming any variant of a product is normalized to zero.
- *Originals.* Each original is produced by a separate producer, at zero marginal cost. All originals are assumed to be (i) of the same quality (indexed by $s_o > 0$) and (ii) perfectly (horizontally) differentiated. Hence, if consumer θ buys a unit of each product in the subset $M \subseteq G$, her utility is given by $m\theta s_o - \sum_{i \in M} p_i$, where $m = |M|$ and p_i is the price charged for product i.
- *Copies.* As for originals, all copies are assumed to be (i) of the same quality (indexed by $0 < s_c < s_o$) and (ii) perfectly (horizontally) differentiated. Regarding their cost, we consider, in the spirit of Johnson (1985), two extreme scenarios: the copying technology involves either a constant unit cost ($c > 0$) and no fixed cost ("*variable copying cost*" model), or a fixed cost ($C > 0$) and no marginal cost ("*fixed copying cost*" model). Supposing that consumer θ copies a unit of each product in the subset $M \subseteq G$, her utility is given by $m\,(\theta s_c - c)$ in the variable copying cost model, and by $m\theta s_c - C$ in the fixed copying cost model.[7]

It is important to note that, in order to focus on the effects of copying, we assume that copying is the only potential source of interdependence between the demands for the various information goods: as just mentioned, the goods are completely differentiated; moreover, we have implicitly assumed that consumers have a sufficient (exogenous) budget to buy all information goods if they so wish.

We examine the variable- and fixed-copying cost models in turn. As will become apparent, the two models lead to very different results. In the former model, the demands for any particular original are completely independent from one another; we can therefore replicate the analysis of the single-good model. On the other hand, as noted by Johnson (1985), the fixed cost of the copying technology introduces some interdependence between the demands for originals: consumers will indeed base their decision to invest in the copying technology on the cost of this technology and on the prices of *all* originals.

[7] A more general formulation for the cost of copying would be: $c(x) = C + cx$ (with $C, c > 0$). It will become apparent below that the model quickly becomes hardly tractable under this general cost function.

3 Multiple goods and variable copying costs

We first analyze the pricing game between an arbitrary number of producers. As will be shown, with perfectly differentiated information goods and variable copying costs, the analysis remains very simple. That allows us to analyze the entry game by incorporating a fixed creation cost. Considering the equilibrium of this two-stage game, we calculate the (long run) welfare implications of copying.

3.1 Pricing game

When the copying technology involves a constant unit cost per copy, it is easily seen that the producers of originals act independently of one another, in accordance with the optimal behavior derived in the single-good model. To see this more clearly, let us define the condition for a typical consumer to buy an original of good i:

$$\begin{aligned}
&\text{consumer } \theta \text{ buys good } i \in G \iff \\
&\theta s_o - p_i + \textstyle\sum_{j \neq i} \max\{\theta s_o - p_j, \theta s_c - c, 0\} \geq \\
&\quad \max\{\theta s_c - c, 0\} + \textstyle\sum_{j \neq i} \max\{\theta s_o - p_j, \theta s_c - c, 0\}.
\end{aligned}$$

In words, the condition says that consumer θ must be better off purchasing good i (and choosing whichever use is the most profitable for the other goods) than copying or not using good i (and still choosing whichever use is the most profitable for the other goods). Because originals are perfectly differentiated and because each copy of an additional good costs the same constant amount, the "whichever use is the most profitable for the other goods" does not depend on which use is made of good i. Therefore, the above condition boils down to $\theta s_o - p_i \geq \max\{\theta s_c - c, 0\}$, which generates the same demand schedule as in the single-good model, as given by expression (2). It follows that, because all producers set the same price, users decide either to buy all information goods or to copy them all (or not to use any). To ease the exposition (and without loss of generality), we set $s_o = 1$ for the rest of this section.

3.2 Entry game

Now, let F_i denote the fixed creation cost faced by producer i. We assume that the cost of creation differs among producers (some producers are more efficient at creating equivalent works than others). Specifically, we assume that F_i is drawn from some cumulative distribution function $H(F)$. This function is assumed to be smooth and increasing on the interval $[\phi, \phi + 1]$, with $0 < \phi < \pi_b = \frac{1}{4} < \phi + 1$, $H(\phi) = 0$ and $H(\phi + 1) = 1$. For given gross profits π, only the producers with $F_i \leq \pi$ will create their information good. Hence, the total number of works created, $n(\pi)$, is endogenously determined as

$$n(\pi) = \begin{cases} H(\pi) & \text{if } \pi \geq \phi, \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, $n(\pi)$ is an increasing function of π. Therefore, we now have a better picture of the social trade-off that copying induces: on the one hand, copying increases social welfare per work (as demonstrated by (5) above) but, on the other hand, copying reduces profits per work and, thereby, the number of works created.

3.3 Welfare effects of copying in the long-run

We now investigate how these two effects balance when copying is either deterred or accommodated. Global welfare (noted Ω) is now defined as welfare per work (i.e., producer's net profit plus consumer surplus) multiplied by the number of works created (if any). To ease the computations, we make the simplifying assumptions that the fixed cost of creation is distributed uniformly and that there is a unit mass of potential producers. We have thus that $n(\pi) = \max\{\pi - \phi, 0\}$. Global welfare is then computed as (with $k = b, d, a$):

$$\begin{aligned} \Omega_k &= \max\left\{\int_{\phi}^{\pi_k} (S_k + \pi_k - F)dF, 0\right\} \\ &= \max\left\{\frac{1}{2}(\pi_k - \phi)(2S_k + \pi_k - \phi), 0\right\}. \end{aligned}$$

Copying is deterred. In the region of parameters where copying is deterred, we find that the difference $\Omega_d - \Omega_b$ is equivalent in sign to $4c^2 - 2s_c(3 - 2\phi) + s_c^2(1 + 2\phi)$. Solving this polynomial for c, we find two positive roots for all admissible values of s_c and ϕ, the large root being larger than $s_c/2$ (above which copying is blockaded). We can therefore conclude that copying deterrence improves global welfare when c is larger than some threshold, $c_d(s_c, \phi)$, which increases with s_c and ϕ.[8]

Copying is accommodated. A similar conclusion is drawn in the region of parameters where copying is accommodated. A few lines of computations establish that *it is only when the relative quality of copies is low enough that copying improves global welfare.* More precisely, for having $\Omega_a > \Omega_b$, c must be larger than some threshold value, $c_a(s_c, \phi)$, which increases with s_c and ϕ. Moreover, if s_c is larger than some lower bound (which decreases with ϕ), $c_a(s_c, \phi)$ is larger than the boundary $s_c(1 - s_c)/(2 - s_c)$ and, therefore, cannot be reached.[9]

[8] Formally, $\Omega_d > \Omega_b$ if and only if $c > c_d(s_c, \phi) \equiv (s_c/4)\left(3 - 2\phi - \sqrt{5 - 20\phi + 4\phi^2}\right)$.

[9] Formally, $c_a(s_c, \phi)$ solves $(2 - s_c)c^4 + 4(1 - s_c)^2 c^3 + 2\left(1 - 3s_c + 3s_c^2 - (4 - s_c)\phi\right)(1 - s_c)c^2 + 4s_c(1 - s_c)^2(s_c + \phi)c - s_c^2(1 - s_c)^2(s_c + 2\phi) = 0$.

The previous results are loosely recorded in the next proposition.

Proposition 3 *When copying involves a constant unit cost and no fixed cost, copying (be it accommodated or deterred) damages welfare in the long run, unless copies are a poor alternative to originals and/or are expensive to acquire.*

The intuition behind Proposition 3 goes as follows. When copies are a poor alternative to originals and/or are expensive to acquire, (actual or threatening) copying erodes only slightly the monopoly power of the producers. Hence, there is only a small reduction in the number of works created, which is more than compensated by the increase in the consumer surplus per work. Yet, the opposite prevails as soon as copies become more attractive. Figure 2 illustrates these results. In areas D_1, D_2 and D_3, producers limit-price to deter copying. Copying deterrence improves global welfare in area D_1, but deteriorates it in areas D_2 and D_3 (worse, in area D_3, the supply of creative works is zero when copying has to be deterred). In areas A_1, A_2 and A_3, producers accommodate copying. Similarly, copying accommodation improves welfare in area A_1, but deteriorates it in areas A_2 and A_3 (with no work created in area A_3). Figure 2 is drawn for $\phi = 0.05$; if we increase ϕ, the curves separating areas A_i and D_i shift up, which reduces the region of parameters where copying has a positive long-run effect on welfare.

[Insert Figure 2 about here]

4 Multiple goods and fixed copying cost

When copying involves a fixed cost rather than variable costs, the demands for originals become interdependent since consumers base their decision to invest in the copying technology on the cost of this technology and on the prices of *all* originals. To see the difference with the previous case, recall the condition for consumer θ to purchase an original of good i. The condition is still that consumer θ must be better off purchasing good i (and choosing whichever use is the most profitable for the other goods) than copying or not using good i (and still choosing whichever use is the most profitable for the other goods). What changes is that the most profitable uses for all other goods does now depend on whether the consumer copies good i or not: if she does, then the cost of copying any number of other goods is zero instead of C.

4.1 User behavior

What is the utility consumer θ can obtain depending on her use of good i? Suppose that $n \geq 2$ goods are available and let p_i denote the price of good i. Since we will be looking for symmetric Bertrand-Nash equilibria, we assume that all other goods are priced the same: $p_j = p\ \forall j \neq i$. To restrict slightly the number of cases to consider, we make the following assumptions:

$$s_c < C < ns_c \,, \tag{6}$$
$$ns_c > s_o \,. \tag{7}$$

Assumption (6) simply says that no consumer will invest in the copying technology if it is to copy only one original ($\theta s_c - C < 0\ \forall\theta$), but that some consumers might invest if it is to copy all n originals ($\exists\theta$ s.t. $\theta ns_c - C > 0$). According to assumption (7), the quality differential between originals and copies is not too large (in particular, n copies are worth at least one original).

We can now determine the most profitable use for the other goods depending on the use made of good i. If the consumer either purchases or does not use good i, it is easily seen that the consumer will treat all the other goods alike: she will either buy, copy or not use them all, leaving her respectively with an additional utility of $(n-1)(\theta s_o - p)$, $(n-1)\theta s_c - C$ or 0.[10] On the other hand, if she copies good i, not using the other goods clearly becomes a dominated option (because the copying technology has been purchased). Hence, the consumer will either purchase or copy all other goods, leaving her respectively with an additional utility of $(n-1)(\theta s_o - p)$ or $(n-1)\theta s_c$. Putting these findings together, we summarize the user's behavior in the following lemma.

Lemma 1 *Facing a price vector $\left(p_i, (p_j = p)_{j\neq i}\right)$ and a copying technology described by (6), a consumer of type θ purchases original i if and only if*

$$\begin{aligned}\theta s_o - p_i + \max\{(n-1)(\theta s_o - p), (n-1)\theta s_c - C, 0\} \\ \geq \max\{(n-1)(\theta s_o - p), n\theta s_c - C, 0\}.\end{aligned} \tag{8}$$

The next logical steps would be, first, to use condition (8) to derive the demand for original i, and next to use the demand function to maximize firm i's profit and, thereby, derive firm i's reaction function. Though feasible, this task turns out to be extremely cumbersome. (We give an idea of the intricacies involved in Appendix 6.1.) We thus renounce to try and give a complete characterization of symmetric Bertrand-Nash equilibria in the model with fixed copying costs. Instead, we provide conditions under which some specific equilibria might (or might not) occur. The price vectors we investigate correspond to the three patterns examined in the model with variable copying costs: blockaded, deterred and accommodated copying. At the end of the section, we emphasize

[10]To see this, let x (resp. y) denote the number of goods $j \neq i$ purchased (resp. copied), with $0 \leq x, y \leq n-1$. If $x \geq 0$, then any situation with $0 < y < n-1-x$ is dominated. Indeed, $y > 0$ implies that the copying technology has been purchased, meaning that copying an additional good makes the consumer strictly better off than not using this good. Now, because the utility from a purchased or a copied good is constant, any situation with $x > 0$ and $y = n-1-x > 0$ is also dominated. We are thus left with three possibilities: (i) $x = n-1, y = 0$, (ii) $x = 0, y = n-1$, and (iii) $x = y = 0$.

the effects of strategic interaction by comparing the symmetric Bertrand-Nash equilibria with the outcomes that would be observed under collusion (or if we were in the presence of a multiproduct monopolist).

4.2 Blockaded copying

As before, copying is blockaded if market conditions are such that copying exerts no threat on producers of originals even when each of them behaves as an unconstrained monopolist. Because, when there is no threat of copying, the demands for the n originals are completely independent of one another, each producer chooses p_i so as to maximize $\pi_i = p_i\left(1 - p_i/s_o\right)$. That is, each firm charges $p_b = s_o/2$. The next proposition states under which condition this behavior constitutes a Nash equilibrium.

Proposition 4 (Blockaded copying) *Each firm charging the monopoly price, $p_b = s_o/2$, is a Nash equilibrium of the game with fixed copying costs if and only if $C \geq (n/2)\, s_c$.*

Proof. See Appendix 6.2. ■

The message of Proposition 4 is clear: *if the most eager consumer needs to copy more than half of the available originals to recoup the fixed cost of the copying technology, then copying exerts no threat on the producers of originals, who can safely charge the monopoly price.*

Let us now turn to the situations where copying *cannot* be blockaded; that is, we assume that $C < (n/2)s_c$. In these situations, copying becomes an actual threat and producers of originals have to decide whether it is more profitable for them to deter or to accommodate copying. As already stressed, the increasing returns to scale in the copying technology transform the choice between copying deterrence or accommodation into a problem of *interdependent* decision making.

4.3 Deterred copying

To deter copying of its product, firm i must find the 'limit price', $\bar{p}_i$, under which all consumers find the original product relatively more attractive than the copy. In the simple model of Sections 2 and 3, firm i could solve this problem in total independence: for consumers to prefer copying to both purchasing and not using, it had to be the case (respectively) that $\theta < (p - c)\,/\,(s_o - s_c)$ and $\theta \geq c/s_c$; clearly, any price below $\bar{p} = cs_o/s_c$ made the joint satisfaction of the two conditions impossible and, thereby, deterred copying.

Now, when copying involves a fixed cost, firm i's limit price will clearly depend on the prices set by the other firms. Intuitively, copying should be harder to deter (in the sense that firm i will have to decrease its price further) the higher the price set by the other firms, and conversely. Indeed, if the other firms set a relatively high price, consumers will have more incentive to invest in the copying technology and, because of increasing returns to scale, they will tend to copy product i along with the other products, unless the price of i is considerably lower.

To formalize the intuition, we first determine firm i's limit price supposing that all other firms charge the same arbitrary price p. That is, we characterize the function $\bar{p}_i(p)$. We then look for a fixed point of this function, $\bar{p}$, and determine under which conditions all firms charging $\bar{p}$ is a Bertrand-Nash equilibrium, in which copying is (collectively) deterred.

4.3.1 Individual limit pricing

Suppose $p_j = p \ \forall j \neq i$. We want to determine the limit price $\bar{p}_i(p)$ under which no consumer finds it profitable to copy product i. Note that firm i is concerned only by deterring the copying of its own product. Yet, as we will see, its behavior will depend on whether consumers copy or not the other products.

Using the analysis of the user behavior (summarized in Lemma 1), let us define the utility for user θ of, respectively, buying or copying product i:

$$\begin{aligned} U_B(\theta, p_i, p) &= \theta s_o - p_i + \underbrace{\max\{(n-1)(\theta s_o - p), (n-1)\theta s_c - C, 0\}}_{MB} \\ U_C(\theta, p_i, p) &= \theta s_c - C + \underbrace{\max\{(n-1)(\theta s_o - p), (n-1)\theta s_c\}}_{MC} \end{aligned}$$

By comparing the exact values of MB and MC, we can express the precise form of the condition $U_B(\theta, p_i, p) \geq U_C(\theta, p_i, p)$ for all configurations of prices and parameters. The next step consists in deriving for which values of p_i the condition is always met in the corresponding region of parameters. Straightforward computations establish the results of this two-step procedure. Table 1 summarizes the results for the case where the other products are relatively expensive–precisely, when $p > s_o C / [(n-1) s_c]$. Table 2 presents the results for the other case. Finally, collecting all these results, we state firm i's limit pricing behavior in Lemma 2.

Users	$U_B(\cdot) \geq U_C(\cdot)$ if	Always met if
$0 \leq \theta \leq \frac{C}{(n-1)s_c}$	$\theta \leq \frac{C-p_i}{ns_c - s_0}$	$p_i \leq \frac{(s_o-s_c)C}{(n-1)s_c}$
$\frac{C}{(n-1)s_c} \leq \theta \leq \frac{(n-1)p-C}{(n-1)(s_o-s_c)}$	$\theta \geq \frac{p_i}{s_o-s_c}$	$p_i \leq \frac{(s_o-s_c)C}{(n-1)s_c}$
$\frac{(n-1)p-C}{(n-1)(s_o-s_c)} \leq \theta \leq \frac{p}{s_o-s_c}$	$\theta \geq \frac{p_i+(n-1)p-C}{n(s_o-s_c)}$	$p_i \leq p - \frac{C}{n-1}$
$\frac{p}{s_o-s_c} \leq \theta \leq 1$	$\theta \geq \frac{p_i-C}{s_o-s_c}$	$p_i \leq p + C$

Table 1: Limit pricing when other products are relatively expensive

Users	$U_B(\cdot) \geq U_C(\cdot)$ if	Always met if
$0 \leq \theta \leq \frac{p}{s_0}$	$\theta \leq \frac{C-p_i}{ns_c-s_0}$	$p_i \leq C - \frac{ns_c-s_o}{s_0}p$
$\frac{p}{s_0} \leq \theta \leq \frac{p}{s_o-s_c}$	$\theta \geq \frac{p_i+(n-1)p-C}{n(s_o-s_c)}$	$p_i \leq C - \frac{ns_c-s_o}{s_0}p$
$\frac{p}{s_o-s_c} \leq \theta \leq 1$	$\theta \geq \frac{p_i-C}{s_o-s_c}$	$p_i \leq p + C$

Table 2: Limit pricing when other products are relatively cheap

Lemma 2 *To deter copying of its product, firm i needs to set its price as follows:*

$$\begin{cases} p_i \leq \bar{p}_i(p) = \frac{(s_o-s_c)C}{(n-1)s_c} & \text{if } p > \frac{s_oC}{(n-1)s_c}, \\ p_i \leq \bar{p}_i(p) = C - \frac{ns_c-s_o}{s_0}p & \text{if } p \leq \frac{s_oC}{(n-1)s_c}. \end{cases}$$

Proof. The proof follows directly from the results summarized in Tables 1 and 2. It is readily checked that the first (resp. second) condition in the lemma is the most stringent among the conditions expressed in Table 1 (resp. in Table 2). ■

Lemma 2 confirms our intuition. When the other products are relatively expensive ($p > s_oC/[(n-1)s_c]$), firm i must price much lower than the other firms ($\bar{p}_i(p) = (s_o - s_c)C/[(n-1)s_c] < p$) in order to discourage copying of its product. On the other hand, as the other products become cheaper, the constraint on i's price relaxes: for $p \leq s_oC/[(n-1)s_c]$, $\bar{p}_i(p)$ decreases with p, and eventually becomes lower than p.

4.3.2 Symmetric limit pricing

The previous findings illustrate how firms tend to free-ride on each other when it comes to deterring copying.[11] The only situation for which no free-riding is observed is when all firms charge the *symmetric limit price* defined by $\bar{p}_i(p) = p$, i.e.,

$$p = p_d \equiv \frac{Cs_o}{ns_c}.$$

This symmetric limit price appears as a likely candidate for an equilibrium with deterred copying. In the next proposition, we state the conditions under which this conjecture proves right. We first define the following threshold:

$$C_d \equiv \frac{n^2s_c(s_o - s_c)}{(n+1)s_o - ns_c}. \tag{9}$$

Note that $C_d < (n/2)s_c \iff ns_c > (n-1)s_o$.

[11] Several authors have studied entry deterrence with several incumbents and investigate whether entry deterrence is a public good. The results they reach are ambiguous. As Applebaum and Weber (1992) summarize, precommitments by an incumbent impose both direct and indirect externalities on other incumbents. This explains why it is generally impossible to determine whether 'investment in deterrence' is too high, or too low relative to the collusive solution.

Proposition 5 (Deterred copying) *Each firm charging the symmetric limit price,* $p_d = (Cs_o)/(ns_c)$*, is a Nash equilibrium of the game with fixed copying costs if and only if* $ns_c > (n-1)s_o$ *and* $C_d \leq C \leq (n/2)s_c$*.*

Proof. See Appendix 6.3. ■

The intuition behind Proposition 5 goes as follows. Suppose the other firms charge the symmetric limit price and consider the "would-be pirates" (i.e., those users for whom $n\theta s_c - C > 0$). We want to determine how those users maximize their utility when they do not purchase product i. Actually, their behavior depends on the relative quality of copies. When the quality of copies is relatively low ($ns_c \leq (n-1)s_o$), the would-be pirates prefer not using i and purchasing all other products, rather than copying all n products. Hence, there is no threat of copying for product i and firm i sees no reason to limit its price. On the other hand, when the quality of copies is relatively high ($ns_c > (n-1)s_o$), the would-be pirates become actual pirates if they decide not to purchase product i. To deter them to do so, firm i must therefore set a low enough price. How low this price should be depends on the fixed copying cost. If this cost is high ($C > (n/2)s_c$), firm i can free-ride on the other firms' effort and set the monopoly price. If the copying cost is low ($C < C_d$), the opposite prevails: firm i must set a limit price below p_d. Finally, for intermediary fixed-copying costs, firm i optimally deters copying by charging the same price as the other firms.

4.3.3 Welfare effects of deterred copying

When the conditions of Proposition 5 are met and n information goods are produced, each producer makes the following (gross) profit:

$$\pi_d(n) = \left(1 - \frac{C}{ns_c}\right)\frac{Cs_o}{ns_c} < \pi_b(n) = \frac{s_o}{4},$$

where $\pi_b(n)$ is the (gross) profit that each producer could obtain if copying did not exist (or could be blockaded). It is easily checked that $\pi_d(n)$ decreases with n, meaning that *deterred copying (under fixed copying costs) is more harmful to producers the larger the number of information goods produced.*

At price $p_d = (Cs_o)/(ns_c)$, (legitimate) users achieve a surplus of

$$S_d(n) = \int_{\frac{C}{ns_c}}^{1} n\left(\theta s_o - \frac{Cs_o}{ns_c}\right)d\theta = \frac{s_o(ns_c - C)^2}{2ns_c^2},$$

which is also equal to total consumer surplus since all other consumers prefer not using any information good. It is straightforward to show that (i) $S_d(n) > S_b(n) = ns_o/8$, and (ii) $S_d(n)$ is an increasing function of n.

Adding total profits to consumer surplus, we derive our measure of social welfare in the short run:

$$W_d(n) = n\pi_d(n) + S_d(n) = \frac{s_o\left(n^2 s_c^2 - C^2\right)}{2ns_c^2}.$$

It turns out that the benefits of copying for the consumers exceeds the losses for the producers. We observe indeed that (i) $W_d(n) > W_b(n) = 3ns_o/8$, and (ii) $W_d(n)$ is an increasing function of n. We therefore conclude that, as in the variable copying cost case, *deterred copying improves social welfare in the short-run* (with respect to a hypothetical economy where copying would be infeasible). The difference with the variable copying cost case is that *welfare improves further as the number of information goods produced increases.*

Because short-run profits decrease with the number of producers, we cannot derive the long-run, free-entry, equilibrium as easily as in the variable-copying cost model.[12] However, we can conjecture that (deterred) copying is likely to have a more detrimental effect on long-run welfare (because, as more goods get produced, the marginal producer faces both higher fixed creation costs and lower gross profits).

4.4 Accommodated copying

We now look for a symmetric Bertrand-Nash equilibrium in which producers find it optimal to tolerate copying. If all originals are priced the same, users will treat all goods alike. That is, the market will be segmented as in the variable-copying cost model: low-θ users will not use any good, intermediate-θ users will copy all goods, and high-θ users will purchase all goods. We need now to determine which common price will achieve such market segmentation.

Suppose that $(n-1)$ firms charge a common price p and that firm i chooses some price p_i in the vicinity of p. To derive the demand facing firm i, we need to identify the user who is indifferent between buying or copying all goods. This user is identified by $\tilde{\theta}$ such that $\tilde{\theta}s_o - p_i + (n-1)(\tilde{\theta}s_o - p) = n\tilde{\theta}s_c - C$; that is

$$\tilde{\theta} = \frac{p_i + (n-1)\,p - C}{n\,(s_o - s_c)}.$$

Because all users with a larger θ then $\tilde{\theta}$ will buy all goods, the demand facing firm i (as long as p_i is not too different from p) is given by

$$D_i(p_i, p) = 1 - \frac{p_i + (n-1)\,p - C}{n\,(s_o - s_c)}. \tag{10}$$

Maximizing $\pi_i(p_i, p) = D_i(p_i, p)p_i$ over p_i yields firm i's reaction function:

$$R_i(p) = \frac{1}{2}\left(n\,(s_o - s_c) - (n-1)\,p + C\right).$$

It is instructive to note that reaction functions are downward sloping. This means that, in the present situation, prices are *strategic substitutes* (using the

[12] Using the same uniform distribution of creation costs as in Section 3, we find now the free-entry number of information goods as the solution to a cubic (instead of linear) equation.

terminology of Bulow *et al.*, 1985). This suggests that, when copying is accommodated, different originals are complements, whereas originals and copies are substitutes.

Our candidate for a symmetric Bertrand-Nash equilibrium with accommodated copying, p_a, must solve $p_a = R_i(p_a)$, which yields

$$p_a = \frac{n\,(s_o - s_c) + C}{n+1}. \tag{11}$$

Naturally, we need now to investigate under which conditions all firms charging p_a is indeed a Nash equilibrium. More precisely, supposing that all other firms charge p_a, we must make sure that firm i has no incentive to set a price that would bring it to a different segment of demand than (10). To do so, we need to determine exactly what the alternative demand segments look like and when they are observed. As before, the starting point is condition (8), which rewrites here as

$$\begin{aligned} &\theta s_o - p_i + R_a \geq L_a, \\ &\text{with } \left\{ \begin{array}{l} R_a \equiv \max\{(n-1)\,(\theta s_o - p_a)\,, (n-1)\,\theta s_c - C, 0\} \\ L_a \equiv \max\{(n-1)\,(\theta s_o - p_a)\,, n\theta s_c - C, 0\}. \end{array} \right. \end{aligned} \tag{12}$$

We start, in the next lemma, by discarding a whole range of cases in which symmetric copying accommodation *cannot* be a Nash equilibrium.

Lemma 3 *If $C \geq C_d$, then all firms charging p_a is not a Bertrand-Nash equilibrium.*

Proof. See Appendix 6.4. ■

The result of Lemma 3 is not surprising. By analogy with the variable-copying cost model, we expect copying accommodation to lead to higher prices than copying deterrence when accommodation is chosen as the most profitable option by the firms. That is, if accommodation is an equilibrium, then $p_a > p_d$ which is equivalent to $C < C_d$. If the opposite is true, copying is costly enough to allow firm i to behave as an unconstrained monopolist when all other firms charge p_a.

Lemma 3 provides a necessary condition (i.e., $C < C_d$) for a symmetric equilibrium with copying accommodation. However, this condition is far from sufficient: additional conditions have to be met to prevent unilateral deviations. Unfortunately, these conditions are very tedious to derive as they depend on the precise configuration of demand (which depends itself on the values of the parameters, in a much more complicated way than for $C \geq C_d$). As a consequence, we shall not attempt here to give a precise characterization of the configurations of parameters where symmetric copying accommodation is a Nash equilibrium. Instead, we will focus on one specific case and use it to illustrate the nature of unilateral deviations from the accommodation price p_a.

Let us consider situations characterized by the following conditions:

$$\begin{cases} s_c < \frac{n-1}{n} s_o, \\ \\ C < \frac{n^2(n-1)s_c(s_o-s_c)}{(n^2-1)s_o+2ns_c} < C_d. \end{cases} \tag{13}$$

Roughly speaking, we take copies as a relatively poor, but inexpensive, alternative to originals.[13] In such situations, it can be shown that the demand function facing firm i when all other firms set the accommodation price p_a is as follows:[14]

$$D_i(p_i, p_a) =$$

$$\begin{cases} D^0 \equiv 0 & \text{for } p_i \geq s_o, \\ D^1 \equiv 1 - \frac{p_i}{s_o} & \text{for } s_o \frac{(n-1)p_a - C}{(n-1)s_o - ns_c} \leq p_i \leq s_o, \\ D^2 \equiv 1 - \frac{p_i + (n-1)p_a - C}{n(s_o - s_c)} & \text{for } p_a - \frac{C}{n-1} \leq p_i \leq s_o \frac{(n-1)p_a - C}{(n-1)s_o - ns_c}, \\ D^3 \equiv 1 - \frac{p_i}{s_o - s_c} & \text{for } \frac{Cs_o}{ns_c} \leq p_i \leq p_a - \frac{C}{n-1}, \\ D^4 \equiv 1 - \frac{p_i}{s_o - s_c} + \frac{C - p_i}{ns_c - s_o} - \frac{p_i}{s_o} & \text{for } \frac{C(s_o - s_c)}{(n-1)s_o} \leq p_i \leq \frac{Cs_o}{ns_c}, \\ D^5 \equiv 1 - \frac{p_i}{s_o} & \text{for } p_i \leq \frac{C(s_o - s_c)}{(n-1)s_o}. \end{cases} \tag{14}$$

Let us shed some light on the various segments composing the demand function. As soon as the price of original i is lower than s_o, some users are willing to purchase the good. In segment D^1, the price is so high that only the most eager (i.e., high θ's) consumers purchase good i. These consumers decide whether to purchase good i or not to use it, given that they purchase anyway all the other, cheaper, information goods.[15] Hence, condition (12) boils down for them to $\theta s_o \geq p_i$. Segment D^2, which corresponds to (10), is obtained when p_i is set in the vicinity of p_a. On top of the previous high θ's consumers, firm i also attracts consumers who prefer purchasing rather than copying all goods. By further decreasing its price, firm i manages to attract some lower-θ consumers. These consumers are resolute to copy all other goods no matter what; but if p_i is sufficiently low, they might prefer the original of good i to the copy. That is, condition (12) writes for these consumers as: $\theta s_o - p_i + (n-1)\,\theta s_c - C \geq n\theta s_c - C \iff \theta\,(s_o - s_c) \geq p_i$, which generates segment D^3. Finally, a further decrease in p_i attracts the very low-θ consumers who decide to purchase and use only good i; segment D^4 (resp. D^5) corresponds to the case where these consumers' second most-preferred option is to copy all goods (resp. not to use any good).

Now, we establish conditions under which $p_i = p_a$ is *not* firm i's best response. We first note that $p_i = p_a$ is the local optimum corresponding to

[13]The condition on C is compatible with our initial requirement that $C > s_c$ if and only if $\left(n^2 - n + 2\right) ns_c < \left(n^2 - n - 1\right)(n-1)\, s_o$, which is itself compatible with the other initial requirement that $ns_c > s_o$ as long as $n > 2$.

[14]The demonstration is available upon request from the author.

[15]The other goods are cheaper because $C < C_d$ implies that $s_o \frac{(n-1)p_a - C}{(n-1)s_o - ns_c} > p_a$.

segment D^2 and yields firm i a profit of[16]

$$\pi_a(n) = \frac{(n(s_o - s_c) + C)^2}{(n+1)^2 n(s_o - s_c)}. \quad (15)$$

This local optimum, however, might not constitute a global optimum. In particular, suppose that firm i deviates by setting a price corresponding to segment D^3. Firm i would maximize its profits under this demand by setting $p_i = (s_o - s_c)/2$. This price is feasible provided that $(s_o - s_c)/2 \geq Cs_o/(ns_c) \iff C \leq ns_c(s_o - s_c)/(2s_o)$. Supposing that the latter condition is met (which itself requires that $n > 3$ to make sure that condition (6) can still be satisfied), firm i's maximum profit over segment D^3 is equal to $\pi^3 = (s_o - s_c)/4$. Some lines of computations establish that the difference $\pi_a(n) - \pi^3$ is equivalent in sign to $4C^2 + 8n(s_o - s_c)C - n(n-1)^2(s_o - s_c)^2$. This quadratic form in C admits two real roots, one positive and one negative. The positive root is equal to $(s_o - s_c)((n+1)\sqrt{n} - 2n)/2$, which can be shown to be larger than $ns_c(s_o - s_c)/(2s_o)$ under conditions (13). It follows that the quadratic form is negative everywhere, meaning that $\pi^3 > \pi_a(n)$ in the situations under review.

We close the discussion by highlighting the intuition behind the previous finding. By setting a price sufficiently lower than p_a, firm i reaches segment D^3 and a larger number of users. In particular, it attracts those consumers who are better off copying all goods rather than purchasing them all, but who would be ready to purchase good i (and still copy all other goods) if it was sufficiently cheaper than the other goods. How cheap good i should be depends on the cost of the copying technology: if this cost is relatively low, copying one less good is no big sacrifice for the users, and a relatively small discount on good i will induce them to buy the good. In such instances, the increase in the number of users makes up for the decrease in the unit price and the deviation is profitable.[17]

4.5 Comparison with the collusive outcome

To emphasize the effects of strategic interaction, let us compare the previous results with what would be observed if the n producers of information goods were able to collude. The cartel of producers would act as a multiproduct monopolist, maximizing its profit by setting a price for all n information products. As in the previous section, we focus on symmetric price vectors.[18] Letting q

[16] It can be checked that (i) $\pi_a(n)$ is a decreasing function of n, and (ii) $C < C_d$ implies that $\pi_a(n) < \pi_b(n) = s_o/4$.

[17] For higher fixed copying costs–i.e., for $C > ns_c(s_o - s_c)/(2s_o)$, firm i's best possible deviation is to set the corner solution price $p_i = Cs_o/(ns_c)$. Yet, it is still possible that this constrained solution yields a higher profit than $\pi_a(n)$.

[18] As the n products are identical, it seems natural to suppose that they are priced the same. However, we leave it to future research to establish that this is indeed an optimal strategy for

denote the common price for the n products, it is easy to develop expression (8) and derive the demand function for any product as

$$D(q)=\begin{cases}0 & \text{if } q\geq s_o-s_c+\frac{C}{n}\\ 1-\frac{nq-C}{n(s_o-s_c)} & \text{if } \frac{Cs_o}{ns_c}\leq q\leq s_o-s_c+\frac{C}{n}\\ 1-\frac{q}{s_0} & \text{if } p\leq\frac{Cs_o}{ns_c}.\end{cases}$$

The cartel therefore faces the following twofold maximization program:

$$\begin{cases}\text{either} & \max_q nq\left(1-\frac{nq-C}{n(s_o-s_c)}\right) \text{ s.t. } \frac{Cs_o}{ns_c}\leq q\leq s_o-s_c+\frac{C}{n},\\ \text{or} & \max_q nq\left(1-\frac{q}{s_0}\right) \text{ s.t. } q\leq\frac{Cs_o}{ns_c}.\end{cases}$$

Recalling expressions (3) and (4), it is quickly seen that this program is equivalent to the single producer's program of Section 2, up to the following change of variable: $C\equiv nc$. The cartel's optimal behavior (with symmetric pricing) can thus be read from Proposition 1:

$$\left|\begin{array}{lll} q_b=\frac{s_o}{2}, & \text{for } \frac{ns_c}{2}\leq C\leq ns_c & \textit{(copying is blockaded)},\\ q_d=\frac{Cs_o}{ns_c}, & \text{for } \frac{ns_c(s_o-s_c)}{2s_o-s_c}\leq C\leq\frac{ns_c}{2} & \textit{(copying is deterred)},\\ q_a=\frac{n(s_o-s_c)+C}{2n}, & \text{for } 0\leq C\leq\frac{ns_c(s_o-s_c)}{2s_o-s_c} & \textit{(copying is accommodated)}.\end{array}\right. \tag{16}$$

Let us now compare the results in (16) with the ones recorded in Propositions 4-5, and in expressions (11) and (15). As far as *blockaded copying* is concerned, there is no difference: $p_b=q_b$ and these prices are optimal in the same region of parameters. This is not surprising as strategic interaction disappears when copying exerts no threat. Regarding *copying deterrence*, strategic interaction introduces a noticeable difference: it does not modify the limit price ($p_d=q_d$) but it narrows the region of parameters where entry deterrence is the optimal conduct. The latter point is established by comparing (9) with (16) and checking that

$$\frac{ns_c\left(s_o-s_c\right)}{2s_o-s_c}<C_d\equiv\frac{n^2s_c(s_o-s_c)}{(n+1)\,s_o-ns_c}.$$

In other words, collusion makes deterrence of copying easier (i.e., optimal for a wider range of costs of the copying technology). This is another illustration of the free-riding problem described above.

Finally, as far as *copying accommodation* is concerned, the impact of strategic interaction appears also quite clearly. A quick comparison of (11) and (16) reveals that $p_a>q_a$. Moreover, computing a firm's profit when copying is collusively accommodated and comparing it to expression (15), one observes that

the multiproduct monopolist.

collusion yields higher profits. To understand why, under strategic interaction, higher prices lead to lower profits, it suffices to recall from the previous section that prices are strategic substitutes under copying accommodation.

5 Conclusion

Information goods fall in the category of *public goods with exclusion*, that is, "public goods the consumption of which by individuals can be controlled, measured and subjected to payment or other contractual limitation" (Drèze, 1980). Exclusion can be achieved through legal authority and/or technical means. However, simply specifying intellectual property laws does not ensure that they will be enforced; similarly, technical protective measures are often imperfect and can be "cracked". As a result, illicit copying (or piracy) cannot be completely avoided. It is therefore extremely important to understand how copying affects the demand for legitimate information goods and the pricing behavior of their producers. It is equally important, for policy purposes, to identify clearly the welfare implications of copying.

This paper addresses these questions within a simple, unified model of competition between originals and copies. We use the vertical differentiation framework proposed by Mussa and Rosen (1978): copies are seen as lower-quality alternatives to originals. In a benchmark model, we consider the market for a single information good. We identify conditions about the relative attractiveness of copies under which the producer either can safely ignore the threat of copying, or has to modify his behavior and decide whether to 'deter' or 'accommodate' copying. In the latter two cases, we show that the competition created by copying enhances social welfare. However, the welfare increase comes at the expense of the producer's profits, which might then be insufficient to cover the (potentially high) fixed cost of creation.

To account for this traditional trade-off between *ex ante* and *ex post* efficiency considerations, we extend the benchmark model by considering an arbitrary number of information goods. We consider two distinct scenarios. In the first scenario, we assume that the copying technology involves a constant unit cost and no fixed cost. Under this assumption, demands for originals are completely independent of one another and we can simply reproduce the results of the benchmark model. Assuming a fixed creation cost that varies through producers, we derive the free-entry number of information goods. We can then balance *ex ante* and *ex post* efficiency considerations and show that copying is likely to damage welfare in the long run (unless copies are a poor alternative to originals and/or are expensive to acquire).

The second scenario assumes that the copying technology involves a positive fixed cost and no marginal cost. The demands for originals are now interdependent because consumers base their decision to copy on the fixed cost of the technology and on the prices of *all* originals. Due to the complexity of the demand system and of the resulting strategic pricing game, we are unable to provide a complete characterization of the set of Bertrand-Nash equilibria. However, we closely examine symmetric equilibria in which copying is either blockaded, deterred or accommodated. We show, in particular, that the latter two equilibria rely on a set of rather restrictive conditions, as the incentives for unilateral deviation are high: producers tend to free-ride (by setting higher prices) when it comes to deterring copying, or they tend to undercut when it comes to accommodating copying.

The directions for future research are twofold. First, and quite obviously, more work needs to be done on the fixed-copying cost model. The characterization of Bertrand-Nash equilibria should be completed. We need not only to characterize all symmetric equilibria in pure strategies, but also to envision asymmetric equilibria and mixed strategies. It is indeed very likely that mixed strategies cannot be dispensed with in this context: because demand functions are discontinuous, payoff functions may fail to be quasi-concave, which may lead to the non-existence of an equilibrium in pure strategies (see Dasgupta and Maskin, 1986).

The second direction for future research consists in exploiting the two versions of the model to address topical issues. For instance, we could try and assess the effects of enhancing technical protective measures for information goods. A case of interest is the so-called "unrippable" CD: because the technical measure seems to decrease the quality of *both* originals and copies (it is claimed that these CDs cannot be copied but, at the same time, legitimate users might not be able to play the CD on the device of their choice), it is not a priori evident that such strategy is profitable.

6 Appendix

6.1 Derivation of demand with fixed copying costs

Let us briefly describe the operations that separate condition (8) from a complete characterization of the demand function for original i.

1. We need first to determine the values of the two maxima for all combinations of parameters. Regarding the LHS, two patterns emerge according to whether p is below or above $\hat{p} \equiv s_o C / ((n-1) s_c)$. Regarding the RHS, four patterns have to be distinguished, depending on the value of p (the threshold being here $\tilde{p} \equiv s_o C / n s_c < \hat{p}$) and on the value of s_c/s_o (the threshold being $(n-1)/n$). Crossing these patterns, we identify six different regimes: 3 cases determined by the value of p ($p > \hat{p}$, $\tilde{p} < p \leq \hat{p}$, and $p < \tilde{p}$) times two cases determined by the value of s_c/s_o (below or above $(n-1)/n$).

2. Next, in each of the six regimes, we need to compare the maxima on the two sides of the inequality. Doing so, we can define intervals for the value of θ inside which condition (8) takes a specific form. For instance, in the regime defined by $p > \hat{p}$ and $s_c/s_o < (n-1)/n$, there are five such different intervals, one of them being the following: for $\theta \in [C/ns_c, C/((n-1)s_c)]$, condition (8) rewrites as $\theta s_o - p_i \geq n\theta s_c - C \iff \theta \geq (C - p_i)/(ns_o - s_c)$.[19]

3. For each interval so defined, the next step consists in establishing conditions on p_i under which the specific form of condition (8) is satisfied. In the previous example, it can be checked that the condition is (i) never met in the interval if $p_i \geq Cs_o/(ns_c)$, and (ii) always met in the interval if $p_i \leq C(s_o - s_c)/((n-1)s_c)$.

4. The last operation consists in collecting all the previous results and to identify, for different ranges of p_i, the corresponding mass of consumers who prefer buying original i. The resulting demand function will typically exhibit a number of kinks (up to 5 in some regimes; see expression (14) for an illustration).

Obviously, this cumbersome process is only a preliminary step towards the characterization of the set of Nash equilibria in prices. Indeed, we should next use the demand function to maximize firm i's profit and, thereby, derive firm i's reaction function. It is easily understood that the combination of several demand regimes and several kinks in the demand function under each regime makes this process even more daunting than the previous one. Furthermore, because demand functions are discontinuous, payoff functions may fail to be quasi-concave, which may lead to the non-existence of an equilibrium in pure strategies (see Dasgupta and Maskin, 1986).

6.2 Proof of Proposition 4

Suppose that $C \geq (n/2)s_c$ and $p_j = p_b = s_o/2\ \forall j \neq i$. Let us show that $p_i = p_b$ is firm i's best response in this case. When all other firms charge $s_o/2$, condition (8) becomes

$$\theta s_o - p_i + R_b \geq L_b,$$
$$\text{with } \begin{cases} R_b \equiv \max\{(n-1)\frac{s_o}{2}(2\theta-1), (n-1)\theta s_c - C, 0\} \\ L_b \equiv \max\{(n-1)\frac{s_o}{2}(2\theta-1), n\theta s_c - C, 0\}. \end{cases} \tag{17}$$

(1) We first establish that L_b cannot be equal to $(n-1)\theta s_c - C$. Indeed, suppose the contrary. Then, we must have (i) $(n-1)\theta s_c - C \geq (n-1)\frac{s_o}{2}(2\theta-1)$

[19] The consumers in this interval decide either to purchase and consume i only, or to copy all products.

$\iff \theta \leq [(n-1)(s_o/2) - C]/[(n-1)(s_o - s_c)]$, and (ii) $(n-1)\theta s_c - C \geq 0$ $\iff \theta \geq C/[(n-1)s_c]$. It is easily checked that the interval on θ defined by the latter two inequalities is non-empty provided that $C < (n-1)s_c/2$, which violates our initial assumption. We thus conclude that L_b equals $(n-1)\frac{s_o}{2}(2\theta-1)$ for $\theta \geq 1/2$, and 0 otherwise.

(2) As for R_b, we first show that $R_b = L_b$ when $(n-1)s_o > ns_c$. In this case, R_b cannot be equal to $n\theta s_c - C$. Indeed, using a similar argument as above, $R_b = n\theta s_c - C$ would necessitate that θ be comprised between $C/(ns_c)$ and $[(n-1)(s_o/2) - C]/[(n-1)s_o - ns_c]$, which is impossible for $C \geq (n/2)s_c$. It follows that $R_b = L_b$ and that consumers buying original i are characterized by $\theta s_o - p_i \geq 0$. Therefore, firm i faces a demand $D_i(p_i, p_b) = (1 - p_i/s_o)$ and its profit-maximizing price is $p_i = p_b$, which completes the proof for this case.

(3) When $(n-1)s_o < ns_c$, R_b can take three values:

$$R_b = \begin{cases} n\theta s_c - C & \text{for } \theta \geq \frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}, \\ (n-1)\frac{s_o}{2}(2\theta-1) & \text{for } \frac{1}{2} \leq \theta \leq \frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}, \\ 0 & \text{for } \theta \leq \frac{1}{2}. \end{cases}$$

Note that, because $\theta \leq 1$, the first option ($R_b = n\theta s_c - C$) is possible only if $C < \bar{C} \equiv ns_c - \frac{n-1}{2}s_o$ (with $\bar{C} > (n/2)s_c$). If the reverse is true, then we have again that $R_b = L_b$ and the proof is completed. Supposing $C < ns_c - \frac{n-1}{2}s_o$, define $p_{12} \equiv C - ns_c + \frac{n+1}{2}s_o$ and $p_{23} \equiv s_o\frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}$. We can summarize the analysis of inequality (17) as follows:

for $\theta \in$	(17) rewrites as	never met if	always met if
$\left]\frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}, 1\right]$	$\theta \geq \frac{p_i+(n-1)(s_o/2)-C}{n(s_o-s_c)}$	$p_i \geq p_{12}$	$p_i \leq p_{23}$
$\left[0, \frac{C-(n-1)(s_o/2)}{ns_c-(n-1)s_o}\right]$	$\theta \geq \frac{p_i}{s_o}$	$p_i \geq p_{23}$	/

We can now write the demand facing firm i:

$$D_i\left(p_i, \frac{s_o}{2}\right) = \begin{cases} 0 & \text{if } p_i \geq p_{12}, \\ 1 - \frac{p_i+(n-1)(s_o/2)-C}{n(s_o-s_c)} & \text{if } p_{23} \leq p_i \leq p_{12}, \\ 1 - \frac{p_i}{s_o} & \text{if } p_i \leq p_{23}. \end{cases}$$

If firm i chooses the third segment of demand, the (unconstrained) optimal price is $p_i = s_o/2$. It is readily checked that this price meets the constraint when $C \geq (n/2)s_c$. This option allows thus the firm to achieve a profit of $\pi_{(3)} = s_o/4$. On the other hand, if firm i chooses the second segment of demand, it can be shown that, with $C \geq (n/2)s_c$, the interior optimum is not feasible. The best firm i can do is to charge $p_i = p_{23}$, which yields a profit of

$$\pi_{(2)} = \frac{s_o(2C-(n-1)s_o)(2ns_c-(n-1)s_o-2C)}{4(ns_c-(n-1)s_o)^2}.$$

Now, because

$$\pi_{(3)} - \pi_{(2)} = \frac{s_o(2C-ns_c)^2}{4(ns_c-(n-1)s_o)^2} > 0,$$

firm i prefers the third segment of demand, which implies that i's bets response is still $p_i = s_o/2$.

6.3 Proof of Proposition 5

Suppose $p_j = \bar{p} = (Cs_o)/(ns_c)\ \forall j \neq i$. We need to determine when $p_i = p_d$ is firm i's best response. We start by deriving the demand function facing firm i, $D_i(p_i, p_d)$. When all other firms charge p_d, condition (8) becomes

$$\theta s_o - p_i + R_d \geq L_d,$$
$$\text{with} \begin{cases} R_d \equiv \max\{(n-1)(\theta s_o - p_d), (n-1)\theta s_c - C, 0\} \\ L_d \equiv \max\{(n-1)(\theta s_o - p_d), n\theta s_c - C, 0\}. \end{cases} \tag{18}$$

Straightforward computations establish that the exact values of MR and ML are as follows:

$*$ for $0 \leq \theta \leq \frac{C}{ns_c}$, $R_d = L_d = 0$,
$*$ for $\frac{C}{ns_c} \leq \theta \leq 1$, $R_d = (n-1)(\theta s_o - p_d)$

$$L_d = \begin{cases} (n-1)(\theta s_o - p_d) & \text{if } ns_c \leq (n-1)s_o, \\ n\theta s_c - C & \text{if } ns_c > (n-1)s_o. \end{cases}$$

If $ns_c \leq (n-1)s_o$, then $R_d = L_d\ \forall\theta$. It follows that condition (18) boils down to $\theta s_o - p_i \geq 0$, which implies that $D_i(p_i, p_d) = 1 - p_i/s_o$, and that firm i's optimal price is $p_i = s_o/2 \neq p_d$. Therefore, for symmetric copying deterrence to be a Nash equilibrium, it is necessary that $ns_c > (n-1)s_o$. In such a case, condition (18) has two possible forms: $\theta s_o - p_i \geq 0$ for $\theta \leq C/(ns_c)$, or $n\theta s_o - p_i - (n-1)p_d \geq n\theta s_c - C$ for $\theta \geq C/(ns_c)$. Computing the conditions on p_i under which each specific form is satisfied in the relevant range, we derive i's demand function as

$$D_i(p_i, \bar{p}) = \begin{cases} 0 & \text{if } p_i \geq n(s_o - s_c) + C - (n-1)p_d, \\ 1 - \frac{p_i + (n-1)p_d - C}{n(s_o - s_c)} & \text{if } p_d \leq p_i \leq n(s_o - s_c) + C - (n-1)p_d, \\ 1 - \frac{p_i}{s_o} & \text{if } p_i \leq p_d. \end{cases}$$

For firm i's best response to be $p_i = p_d$, the interior solutions when i's maximizes over the second or third segments of demand must be infeasible. As for the second segment, the unconstrained optimum is $p_i = p_{(2)} \equiv (1/2)(n(s_o - s_c) + C - (n-1)p_d)$; one easily checks that $p_{(2)} \leq p_d$ for $C \geq C_d$. As for the third segment, the unconstrained optimum is $p_i = s_o/2$, which is clearly larger than p_d for $C \leq (n/2)s_c$. This completes the proof.

6.4 Proof of Lemma 3

The first thing to note is that $C \geq C_d \iff p_a \leq (s_oC)/(ns_c)$. In this case, the comparison between R_a and L_a is rather simple. First, if $(n-1)s_c > ns_o$, it is easily checked that $R_a = L_a \ \forall\theta$. It follows that condition (12) boils down to $\theta s_o - p_i \geq 0$, and we know that firm i's best reply is then $p_i = s_o/2 \neq p_a$, which establishes our result for this particular case. Second, if $(n-1)\, s_c < ns_o$, the analysis of condition (12) can be summarized by the following table (based on straightforward computations):

for $\theta \in$	(12) rewrites as	never met if	always met if
$\left[\frac{C-(n-1)p_a}{ns_c-(n-1)s_o}, 1\right]$	$\theta \geq \frac{p_i+(n-1)p_a-C}{n(s_o-s_c)}$	$p_i \geq 2p_a$	$p_i \leq s_o\frac{C-(n-1)p_a}{ns_c-(n-1)s_o}$
$\left[0, \frac{C-(n-1)p_a}{ns_c-(n-1)s_o}\right]$	$\theta \geq \frac{p_i}{s_o}$	$p_i \geq s_o\frac{C-(n-1)p_a}{ns_c-(n-1)s_o}$	/

We can now write the demand facing firm i:

$$D_i\left(p_i, p_a\right) = \begin{cases} 0 & \text{if } p_i \geq 2p_a, \\ 1 - \frac{p_i+(n-1)p_a-C}{n(s_o-s_c)} & \text{if } s_o\frac{C-(n-1)p_a}{ns_c-(n-1)s_o} \leq p_i \leq 2p_a, \\ 1 - \frac{p_i}{s_o} & \text{if } p_i \leq s_o\frac{C-(n-1)p_a}{ns_c-(n-1)s_o}. \end{cases}$$

For p_a to be firm i's best reply, it must be the case that firm i selects the second segment of demand and that the interior solution to its profit-maximization problem be feasible. This is so provided that

$$p_a \geq s_o\frac{C-(n-1)\,p_a}{ns_c-(n-1)\,s_o} \iff p_a \geq \frac{s_oC}{ns_c},$$

which contradicts our initial assumption and completes the proof.

References

[1] Applebaum, E. and Weber, S. (1992). A note on the free rider problem in oligopoly. *Economics Letters* **40**: 473-480.

[2] Bain, J. (1956). *Barriers to new competition.* Harvard Business Press, Cambridge (Mass.).

[3] Ben-Shahar, D. and Jacob, A. (2001). Preach for a breach: Selective enforcement of copyrights as an optimal monopolistic behavior. Mimeo, Arison School of Business, Israel. Available at http://www.faculty.idc.ac.il/ben-shahar/papers/Preach%20for%20a%20Breach%2020.07.01.pdf

[4] Besen, S.M. and Kirby, S.N. (1989). Private copying, appropriability, and optimal copying royalties. *Journal of Law and Economics* **32**: 255-280.

[5] Besen, S.M. and Raskind, L.J. (1991). An introduction to the law and economics of intellectual property. *Journal of Economic Perspectives* **5**: 3-27.

[6] Bulow, J., Geanakoplos, J. and Klemperer, P. (1985). Multimarket oligopoly: Strategic substitutes and complements. *Journal of Political Economy* **93**: 488-511.

[7] Chen, Y. and Png, I. (2001). Software pricing and copyright enforcement. Mimeo, National University of Singapore. Available at http://www.comp.nus.edu.sg/~ipng/research/copy_is1.pdf

[8] Conner, K.R. and Rumelt, R.P. (1991). Software copying: An analysis of protection strategies. *Management Science* **37**: 125-139.

[9] Dasgupta, P. and Maskin, E. (1986). The existence of equilibrium in discontinuous economic games. I: Theory. *Review of Economic Studies* **53**: 1-26.

[10] Drèze, J. (1980). Public goods with exclusion. *Journal of Public Economics* 13: 15-24.

[11] Duchêne, A. and Waelbroeck, P. (2001). Welfare implications of illegal copies: the case of peer-to-peer distribution technologies. Mimeo, CERAS-ENPC, Paris. Available at http://homepages.ulb.ac.be/~pwaelbro/paper11.pdf

[12] Gayer, A. and Shy, O. (2001a). Copyright protection and hardware taxation. Mimeo, University of Haifa. Available at http://econ.haifa.ac.il/~ozshy/copyingtax15.pdf

[13] Gayer, A. and Shy, O. (2001b). Internet, peer-to-peer, and intellectual property in markets for digital products. Mimeo, University of Haifa. Available at http://econ.haifa.ac.il/~ozshy/freeware18.pdf

[14] Harbaugh, R. and Khemka, R. (2001). Does copyright enforcement encourage copying? Mimeo, Claremont McKenna College. Available at http://econ.mckenna.edu/papers/2000-14.pdf

[15] Hui, K.L., Png, I., and Cui, Y. (2001). Copying and the legitimate demand for recorded music. Mimeo, National University of Singapore. Available at http://www.comp.nus.edu.sg/~ipng/research/copying_text_2.pdf

[16] Johnson, W.R. (1985). The economics of copying. *Journal of Political Economy* **93**: 158-174.

[17] Landes, W.M. and Posner, R.A. (1989). An economic analysis of copyright law. *Journal of Legal Studies* **38**: 325-363.

[18] Liebowitz, S.J. (1985). Copying and indirect appropriability: Photocopying of journals. *Journal of Political Economy* **93**: 945-957.

[19] Novos, I.E. and Waldman, M. (1984). The effects of increased copyright protection: An analytical approach. *Journal of Political Economy* **92**: 236-246.

[20] Mussa, M., and Rosen, S. (1978). Monopoly and product quality. *Journal of Economic Theory* **18**: 301-317.

[21] Plant, A. (1934). The economic aspects of copyright in books. *Economica* **1**: 167-195.

[22] Shy, O. and Thisse, J.-F. (1999). A strategic approach to software protection. *Journal of Economics and Management Strategy* **8**: 163-190.

[23] Takeyama, L.N. (1994). The welfare implications of unauthorized reproduction of intellectual property in the presence of network externalities. *Journal of Industrial Economics* **62**: 155-166.

[24] Varian, H.R. (1998). Markets for information goods. Mimeo, University of California, Berkeley. Available at http://www.sims.berkeley.edu/~hal/Papers/japan/japan.pdf

[25] Watt, R. (2000). *Copyright and Economic Theory. Friends or Foes?* Edward Elgar Publishing, Cheltenham (UK) and Northampton (Mass.).

[26] Yoon, K. (2001). The optimal level of copyright protection. Mimeo, Department of Economics, Korea University. Available at http://econ.korea.ac.kr/~kiho/works/cp2_dist.pdf

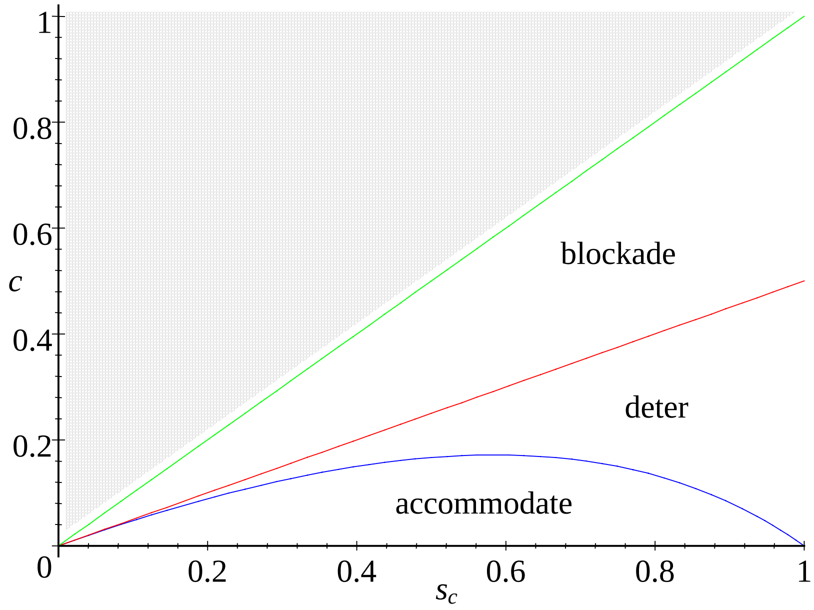

Figure 1. Producer's optimal strategy in the single-good model.

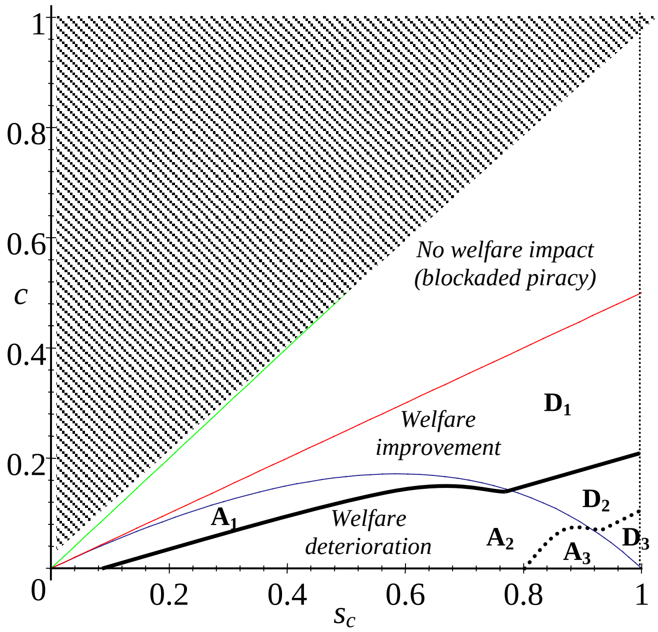

Figure 2. Long-run welfare effects in the variable copying cost model.

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