



## **Abstract**

We compare the advertising intensity and content of programming in a market with competing media platforms. With pay-tv media platforms have two sources of revenues, advertising revenues and revenues from viewers. With free-to-air media platforms receive all revenues from advertising. We show that if viewers strongly dislike advertising, the advertising intensity is greater under free-to-air television. We also show that free-to-air television tends to provide more similar content whereas pay-tv stations differentiate their content. In addition, we compare the welfare properties of the two different schemes.

JEL-Classification: D43, L13, L82

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# 1 Introduction

[very preliminary and incomplete]

With the appearance of decoders viewers can be charged for their consumption of certain programming. This possibility to charge viewers affects the advertising and content decisions of media platforms. In this paper, we present a formal model which allows us to compare content and advertising decisions in the conventional world of free-to-air television as opposed to the new world of pay-tv.

Obviously, a media platform can only succeed if it has viewers. Otherwise, its revenues from advertising as well as its revenues from charging viewers would be zero. Under free-to-air, advertising is the only source of revenue, whereas under pay-tv viewers can be charged directly. Advertising is typically a nuisance for viewers. Therefore, the amount of advertising constitutes an indirect charge to consumers. Viewers are interested in programming with little advertising; hence advertisers exert an external effect on viewers. Conversely, advertisers are interested in a large number of viewers; hence viewers exert an external effect on advertisers. Therefore, the market constitutes a so-called two-sided market and platforms compete for viewers and advertisers.

In this market media platforms are assumed to control the advertising space they offer. This implies that they control the external effect exerted on viewer. Viewers, however, freely choose which program to choose so that platforms cannot directly control their number of viewers. With pay-tv platforms have two instruments to charge viewers; they can charge directly and they can impose a disutility through the amount of advertising they choose. We show that equilibrium profits are independent of how much nuisance advertising generates to viewers. This neutrality does not hold under free-to-air, here profits are smaller if the nuisance advertising generates to viewers is greater.

**Related Literature.** A series of papers has analyzed the provision of content and advertising in media markets. Closest to our paper is the work by Anderson and Coate (2003), Dukes and Gal-or (2003a), Gabszewicz, Laussel, and Sonnac (2001, 2002a, 2004), and Reisinger (2004).

Anderson and Coate (2003) consider a model of two competing media platforms with given content. They focus on the welfare properties of the equilibrium in which platforms only charge advertisers. A large number of advertisers, which offer their products as monopolists, can advertise on none, one, or both platform. Advertising generates rents because it provides information to consumers but is a nuisance for viewers.<sup>1</sup> The natural question therefore is to ask whether the market over- or underprovides advertising. Anderson and Coate show that the equilibrium advertising level is below the optimal one if the nuisance of advertising is small and above it if the nuisance is large. In addition, they show that underprovision is more likely if viewers

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<sup>1</sup>For an excellent overview on the economics of advertising see Bagwell (2003). He elaborates on the welfare properties of informative advertising compared to other forms of advertising.

perceive platforms as closer substitutes because competition for viewers becomes more intense to the effect that platforms charge more for advertising.

Reisinger (2004), like Anderson and Coate (2003), considers a model in which content is given. However, he postulates that platforms compete for advertisers; more specifically, he assumes single-homing so that advertisers choose on which platform to advertise. He shows that if media platforms cannot charge viewers, closer substitutes of platforms for viewers can lead to higher profits because competition on the advertisers side is reduced.

In Gabszewicz, Laussel, and Sonnac (2001, 2002a, 2004) and Dukes and Gal-or (2003) platforms choose content at a first stage. In Gabszewicz, Laussel, and Sonnac (2001, 2002a), similar to Anderson and Coate (2003), a large number of advertisers can advertise on none, one, or both platform, but viewers are assumed to be indifferent about the level of advertising. This implies that the inter-group externalities between advertisers and viewers exist only in one direction, namely that advertisers like a media platform with many viewers. Gabszewicz, Laussel, and Sonnac then show that when viewers are not charged both platforms provide the same content, that is, there is minimal differentiation in the content space. However, when viewers are charged there is maximal differentiation in the content space. In the latter case, the logic is the same as in D'Aspremont, Gabszewicz, and Thisse (1979) because more differentiation reduces competition for viewers and this effect dominates. In the former case, platforms compete for viewers because more viewers lead to larger advertising revenues. This is reminiscent of the Hotelling model in which prices are fixed so that providing the same content at the center of the content space is the only equilibrium.

In Gabszewicz, Laussel, and Sonnac (2004) viewers select their optimal program mix by selecting among two channels and viewers dislike advertising. Media platforms set content and advertising levels; advertisers are homogeneous. Because of the latter assumption advertising rates per viewer are constant. Instead of setting prices to viewers as in the standard Hotelling model platforms set disutilities from advertising. In equilibrium, platforms choose maximal differentiation in the content space if the disutility from advertising is linear in the amount of advertising.

Dukes and Gal-or (2003a) present a different, elaborate model in which two advertisers compete for viewers who are also the consumers of the products. More advertising increases the probability that a consumer becomes informed (as in Grossman and Shapiro, 1984). This implies that less advertising by the two advertisers leads to higher prices because each advertiser has a larger captive segment. Differentiation among viewers is with respect to the product characteristics of the advertised products and with respect to media content. Viewers select their optimal mix of programming as in Gabszewicz, Laussel, and Sonnac (2004). Product characteristics are assumed to be given, but, at the first stage of the game, platforms decide on the content they offer. At the second stage, platforms and advertisers reach agreements on the amount of advertising and advertisers set prices for their products and choose advertising levels. Here,

Dukes and Gal-or assume that any surplus generated by an advertiser-platform pair is evenly split among the two parties. Different from Gabszewicz, Laussel, and Sonnac (2004) they obtain that both platforms offer the same content. The intuition for their results is the following; first, when a platform locates closer to the center it increases the number of viewers that will exclusively consume content from this platform (this effect is also present in Gabszewicz, Laussel, and Sonnac, 2004). Moving closer, however, also gives rise to a competition effect. In Dukes and Galor (2003a) less differentiation increases the platforms' profits because under program duplication advertisers choose a low level of advertising together with high prices leading to high revenues. Since any surplus is shared between advertiser and platform, platforms benefit from relaxed competition among advertisers in the product market.

More generally, our paper is related to the growing literature on two-sided markets (see e.g. Armstrong, 2004, Evans, 2004, Rochet and Tirole, 2003, 2004). Most closely to our work, Armstrong (2004, section 5) analyzes advertising competition among media platforms for given content, where advertisers can advertise on both platforms and platforms set prices on both sides of the market.<sup>2</sup> He shows that there is always underprovision of advertising compared to the social optimum. In addition, if platform set per-consumer advertising charges, a platform's revenues from advertising are passed onto consumers in the form of lower prices. Profits decrease if platforms are closer substitutes.

**Our contribution and plan of the paper.** As the above papers we present a model of competing media platforms. Our model is similar to Armstrong (2004) and Anderson and Coate (2003), with the important difference that content is determined by the media platforms themselves (as in Gabszewicz, Laussel, and Sonnac, 2001, 2002a, 2004). We distinguish between pay-tv, where advertisers and viewers have to pay, and free-to-air tv, where only advertisers have to pay. Our model is presented in section 2. Section 3 contains our analysis of pay-tv — this is partly a restatement of section 5 in Armstrong. Section 4 contains the analysis of free-to-air tv — for given content this resembles the analysis of Anderson and Coate (2003). In both cases, pay-tv and free-to-air tv we endogenize the choice of content as in Gabszewicz, Laussel, and Sonnac (2001, 2002a). We show that their minimum differentiation result relies on their assumption that consumers are indifferent about advertising. Depending on the nuisance of advertising content is between minimum and maximum differentiation. In section 5 we compare equilibrium content and advertising under pay tv and free-to-air tv, depending on the potential differentiation between platforms and the nuisance from advertising. We also compare welfare under pay-tv and free-to-air when content is given and when content is chosen by platforms. Section 6 concludes.

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<sup>2</sup>This analysis is closely related to Rysman (2002) and Rochet and Tirole (2003, section 5).

## 2 The Model and Social Optimum

**Viewers.** Viewers consume advertising and content from either one of two programs. They constitute the buyer side in the market. The buyer side is of mass  $N$ . A buyer of type  $\beta$  has a particular taste for programming. Programming can be in the  $[0, 1]$ -interval. A buyer has preference parameter  $\beta \in [0, 1]$ , which reflects her favorite type of programming. If a program is located at  $d_1$  a buyer incurs a disutility  $\tau(\beta - d_1)^2$ , where  $\tau > 0$  is the disutility parameter from consuming programming content that does not satisfy a buyer  $\beta$ 's tastes. A program also contains advertising. We assume that advertising and content are additively separable in the utility function.<sup>3</sup> Advertising of a program also leads to a utility loss, that is, viewers are assumed to dislike advertising. The corresponding utility loss is assumed to be  $\delta a_i$ , where  $\delta$  is the disutility parameter for advertising and  $a_i$  is the amount of advertising. All consumers are assumed to have the same parameter  $\delta$ . A viewer has to pay a fee  $s_i$  for programming. This either is pay-per-view if we consider competition between programs or a subscription fee for a channel if we consider competition between channels. The indirect utility of a viewer of type  $\beta$  from consuming program 1 is

$$v - \delta a_1 - \tau(\beta - d_1)^2 - s_1$$

where  $v$  is the willingness-to-pay for perfect programming and zero advertising. We implicitly assume that  $v$  is sufficiently large such that all potential viewers view one program, that is, we have full market coverage. Similarly, for program 2. We also assume that viewers cannot mix two programs if they prefer a mix – this means we are considering competition for a particular time slot rather than competition between two channels.<sup>4</sup> If the two programs are located at  $d_1$  and  $1 - d_2$  on the line, there is a viewer  $b_1$  who is indifferent between the two programs,

$$-\delta a_1 - s_1 - \tau(b_1 - d_1)^2 = -\delta a_2 - s_2 - \tau((1 - b_1) - d_2)^2.$$

Solving for  $b_1$  one obtains

$$b_1 = \frac{d_1 + 1 - d_2}{2} - \frac{\delta(a_1 - a_2) + (s_1 - s_2)}{2(1 - d_1 - d_2)\tau} \quad (1)$$

All viewers to the left of  $b_1$  view program 1 and all viewers to the right of  $b_1$  view program 2. Hence, the total number of viewers of program 1 is  $Nb_1$  and the total number of viewers of program 2 is  $Nb_2$  where  $b_2 \equiv 1 - b_1$ .

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<sup>3</sup>This assumption is also made in the cited literature above. It clearly is a restrictive assumption but avoids additional computational problems.

<sup>4</sup>Dukes and Gal-or (2003a) and Gabszewicz, Laussel and Sonnac (2004) consider channel competition so that viewers can mix. The resulting demand, however, resembles demand derived in the present model.

**Advertisers.** Advertisers of mass 1 sell products to viewers who are also the consumers of the products. Products are produced at constant marginal costs, which without loss of generality are set equal to zero. A product is produced at quality  $\alpha$  and consumers have willingness to pay  $\alpha$  for a good of quality  $\alpha$ . Each producer has monopoly power and can therefore extract the full surplus from consumers, that is, a product of quality  $\alpha$  is sold at price  $\alpha$ . Producers differ with respect to the quality of the good they offer. Quality is distributed on some interval  $[0, \alpha^{\max}]$  according to a p.d.f.  $F$  with  $F(0) = 0$  and is assumed to have a density which is continuously differentiable. As our lead example, we consider the case that  $F$  is uniform on  $[0, 1]$ .

Advertisers can only sell to those consumers which have seen the ad. If an advertiser advertises in a particular program it is assumed that all viewers of this program are aware of the corresponding product. Advertisers can advertise in none, one, or both programs.<sup>5</sup> Advertisers have to pay the advertising charge  $r_i$ . The profit for advertiser  $\alpha$  from advertising in program  $i$  is

$$N\alpha b_i - r_i$$

The marginal advertiser for program  $i$ ,  $\underline{\alpha}_i = r_i/(Nb_i)$ , makes zero profit. Hence the amount of advertising in program  $i$  is

$$a_i = 1 - F\left(\frac{r_i}{Nb_i}\right). \quad (2)$$

The advertising space  $a_i$  determines the advertising charge per viewer  $r_i/(Nb_i)$ ; it is not affected by the decisions of the competing platform.

**Media Platform.** Media platform  $i$  invites advertisers to its platform. For this it provides advertising space  $a_i$  and attracts a number  $b_i$  of customers. We distinguish between two different technologies,

- free-to-air and
- pay tv.

Under free-to-air a program is provided for free to customers. With pay-tv customers are charged a pay-per-view price  $s_i$ . Here, we allow for the subsidization of customers, that is,  $s_i$  can take negative values. Under free-to-air the price  $s_i$  is fixed equal to zero. Profit of program  $i$  is

$$\pi_i = Nb_i s_i + a_i r_i.$$

**The Content, Pricing and Advertising Game.** The two media platforms are the non-atomistic players in this game. They play a three-stage game. In the first stage,

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<sup>5</sup>In the terminology of the literature on two-sided markets, the viewers' market is a competitive bottleneck, see Armstrong (2004).

$i = 1, 2$	platform $i$
$\alpha$	type of advertiser
$F$	distribution of advertisers' profits per customer
$a_i$	amount of advertising on platform $i$
$r_i$	advertising charge
$d_i$	location of platform
$\beta$	type of buyer
$b_i$	number of buyers at platform $i$
$s_i$	pay-per-view price
$N$	mass of customers
$\tau$	disutility parameter for content misspecification
$\delta$	disutility parameter for advertising

Table 1: Notation

they determine the content of the platform. In the second stage, they determine the space for advertising and set the subscription or pay-per-view price. In the third stage, advertisers decide where to advertise and viewers decide which channel to view. At this stage, advertisers exert a negative externality on viewers. Hence, advertisers and viewers play an anonymous game. We can summarize the game as follows.

- Stage 1: Media platforms simultaneously decide on content  $d_i$ .
- Stage 2: Media platforms simultaneously decide on advertising space  $a_i$  and pay-per-view prices  $s_i$ , where  $s_i = 0$ ,  $i = 1, 2$ , under free-to-air.
- Stage 3: Advertisers and viewers simultaneously take their decisions: advertisers place their ads in none, one, or both programs and viewers view program 1 or program 2.

We characterize subgame perfect Nash equilibria of this game. Note that in stage 2, media platforms are quantity setters in the advertising market.<sup>6</sup> Advertising charges  $r_i$  and  $r_j$  clear the market. Since  $F$  is invertible on the support of  $\alpha$  equation (2) can be rewritten which gives expressions for the advertising charges

$$r_i = Nb_i F^{-1}(1 - a_i).$$

Our notation is summarized in Table 1.

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<sup>6</sup>Since advertising space fully determines the advertising rate per viewer (see the discussion of equation (2)) it does not matter whether platforms set advertising space or rate per viewer. However, setting a rate per viewer requires that platforms are able to monitor the number of viewers.

**Welfare.** In our model with full viewer participation only the amount of advertising and the content of programs affect welfare. Hence welfare is the sum of a constant  $K = Nv$ , welfare with respect to content, and welfare with respect to advertising,  $W = K + W^{co} + W^{ad}$ . Welfare with respect to content is

$$W^{co} = -N\tau \int_0^{b_1} (\beta - d_1)^2 d\beta - N\tau \int_0^{b_2} (\beta - d_2)^2 d\beta.$$

Welfare with respect to advertising is

$$W^{ad} = Nb_1 \int_{\min\{\underline{\alpha}_1, \alpha^{\max}\}}^{\alpha^{\max}} (\alpha - \delta) dF(\alpha) + Nb_2 \int_{\min\{\underline{\alpha}_2, \alpha^{\max}\}}^{\alpha^{\max}} (\alpha - \delta) dF(\alpha)$$

**Social Optimum.** Welfare is maximized with respect to content when  $d_1^W = d_2^W = 1/4$  – these are the locations which minimize transportation costs if transportation costs are strictly convex as it is the case for our quadratic specification. Consider now welfare with respect to advertising. The derivative of  $W^{ad}$  with respect to  $\underline{\alpha}_i$  is

$$\frac{\partial W^{ad}}{\partial \underline{\alpha}_i} = -Nb_i(\underline{\alpha}_i - \delta)f(\underline{\alpha}_i).$$

If  $\delta < \alpha^{\max}$  the welfare maximizing number of advertisers is determined by  $\underline{\alpha}_i = \delta$  so that  $a_i^W = 1 - F(\delta)$ . Otherwise,  $a_i^W = 0$ . In the special case that  $F$  is uniform on  $[0, 1]$ ,  $a_i^W = \max\{1 - \delta, 0\}$ .

### 3 Pay-tv

The solution to the system of equations (1), (2) for  $i = 1, 2$  and  $b_2 = 1 - b_1$  is the equilibrium in stage 3. This determines how advertising charges react to pay-per-view prices  $s_i$  and to advertising levels  $a_i$ , which are set in stage 2.

**Equilibrium for given program content.**<sup>7</sup> The equilibrium is characterized by the system of four first-order conditions

$$\frac{\partial \pi_i}{\partial s_i} = N \left( b_i + \frac{\partial b_i}{\partial s_i} s_i \right) + a_i \frac{\partial r_i}{\partial s_i} = 0, \quad i = 1, 2 \quad (3)$$

$$\frac{\partial \pi_i}{\partial a_i} = N \frac{\partial b_i}{\partial a_i} s_i + r_i + a_i \frac{\partial r_i}{\partial a_i} \leq 0, \quad i = 1, 2 \quad (4)$$

First-order conditions (3) must hold with equality because we allow for subsidies to viewers. In contrast, the amount of advertising cannot be negative. Therefore, first-order conditions (4) hold with equality for interior solutions.

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<sup>7</sup>This part of the analysis with exogenous content is related to section 6.1 in Anderson and Coate (2003) and section 5.2 in Armstrong (2004).

The advertising revenue per viewer is  $\rho(a_i) \equiv a_i F^{-1}(1 - a_i)$ . We need that this function is single-peaked, which is implied by the following assumption.

**Assumption.** *The advertising revenue per viewer  $\rho$  is concave in  $a_i$ .*

Note that in the uniform case  $\rho(a_i) = a_i(1 - a_i)$ , which is concave. Rewriting the first-order conditions for platform  $i$ , one obtains

$$b_i + [s_i + \rho(a_i)] \frac{\partial b_i}{\partial s_i} = 0, \quad (5)$$

$$\rho'(a_i) b_i + [s_i + \rho(a_i)] \frac{\partial b_i}{\partial a_i} \leq 0. \quad (6)$$

These two first-order conditions determine platform  $i$ 's amount of advertising independent of content, own pay-per-view price and the competing platform's decisions, as stated in the following lemma.

**Lemma 1** *Media Platform  $i$ 's profit maximizing amount of advertising is a constant. It is determined by*

$$\rho'(a_i) = \delta \quad (7)$$

if  $\delta < \alpha^{\max}$ . Otherwise it is 0.

**Proof.** Since  $\partial b_i / \partial a_i = \delta \partial b_i / \partial s_i$  inequality (6) can be rewritten as

$$\frac{\rho'(a_i)}{\delta} b_i + [s_i + \rho(a_i)] \frac{\partial b_i}{\partial s_i} \leq 0$$

If this condition is satisfied with equality so that  $a_i$  is positive, this condition together with (5) implies (7). Since  $\rho$  is concave, the best-response  $a_i$  is uniquely determined by (7). When  $\delta \geq \alpha^{\max}$  the condition holds with strict inequality, the platform is constrained by the fact that advertising space cannot be negative. Hence,  $a_i = 0$ , in this case the solution coincides with the social optimum. ■

Providing a fixed advertising space, which depends on the disutility parameter for advertising, is a best-response property of each media platform. As the disutility parameter becomes smaller, the profit maximizing advertising space  $a_i$  increases. In the absence of the viewers' reaction to advertising, i.e.  $\delta = 0$ , each media platform provides the monopoly advertising space  $\rho'(a_i) = 0$ .

In the uniform case advertising is  $a_i = \max\{(1 - \delta)/2, 0\}$ . For a sufficiently high disutility from advertising, in the example  $\delta > 1$ , the social optimum is implemented, which is zero advertising. For a sufficiently low disutility from advertising, there is an underprovision of advertising. The reason is that platforms cannot absorb all rents from

advertisers. In general, we have that in pay-tv markets there is always underprovision of advertising until the market is shut down by the media platforms.

To determine the pay-per-view price we solve the two first-order conditions (3) for given advertising space  $a_i$ ,  $i = 1, 2$ .

$$s_i = \frac{(1 - d_i - d_j)(3 + d_i - d_j)}{3}\tau + \frac{\delta(a_j - a_i)}{3} - \frac{2\rho(a_i) + \rho(a_j)}{3}$$

The right-hand side of the expression which determines  $s_i$  consists of three terms: the standard Hotelling-term and two terms that depend on advertising. The first of those two latter terms captures the viewers' disutility from ads. If platform  $i$  admits more advertising than the competing platform it has to compensate its own subscribers via a lower pay-per-view price. In other words, if advertising was given exogenously, more advertising on platform  $i$  leads to an asymmetry between platforms and the equilibrium price reaction is for platform  $i$  to price more aggressively and for platform  $j$  to price less aggressively. The second of those two latter terms reflects the role of advertising to generate revenues. The higher the advertising revenues per viewer  $\rho$  the more attractive viewers are. Hence, prices are lowered to subsidize viewers. This illustrates that the platform can adjust the pay-per-view price to the advertising space it provides.

As implied by Lemma 1, both firms provide the same advertising space,  $a_1 = a_2$ . Hence, in the equilibrium of stage 2

$$s_i = \frac{(1 - d_i - d_j)(3 + d_i - d_j)}{3}\tau - \rho(a_i).$$

There is a full pass-through of advertising revenues into lower pay-per-view prices. An immediate consequence is that the advertising revenues do not affect equilibrium profits of the two platforms (profit neutrality). Notice that the disutility parameter for ads,  $\delta$ , does not enter directly into the expression for equilibrium prices. However, it affects prices indirectly through the advertising space  $a_i$ .

In the uniform case, we can write the equilibrium pay-per-view prices as

$$s_i = \frac{(1 - d_i - d_j)(3 + d_i - d_j)}{3}\tau - \frac{(1 + \delta)}{2} \max \left\{ \frac{1 - \delta}{2}, 0 \right\}$$

so that  $s_i$  is increasing in  $\delta$  for  $\delta < 1$  (this reflects the pass-through result as advertising revenues are maximized for  $\delta = 0$  and decrease for higher values of  $\delta$ ), and constant in  $\delta$  for  $\delta > 1$ . Equilibrium profits are

$$\pi_i = (1 - d_i - d_j) \frac{(3 + d_i - d_j)^2}{18} \tau N$$

**Remark 1** *The result on equilibrium advertising space and viewers' prices has an analogy with two-part pricing. In our model, advertisers are the "sticky" part of the market*

since viewers are competitive bottlenecks. Platforms have two instruments at their disposal to attract viewers, advertising space and subscription prices. As with two-part prices, platforms set the advertising space that maximize the joint surplus of the platform and its viewers. It then extracts part of this joint surplus using the fixed subscription fee according to the intensity of competition with the rival platform. This explains why the platform decides to shut down the advertising market when  $\delta$  is higher than  $\alpha^{\max}$ : the disutility for viewers always exceeds any profit that could be made from advertisers. It also explains why, for lower values of  $\delta$ , the solution is given by (7) and there is always under-provision of advertising: the platform takes into account only the surplus of its viewers but not the advertisers' and sets the advertising space that equates at the margin the revenue per viewer to its disutility.

**The Provision of Program Content.** Because of profit neutrality the analysis at the first stage reduces to the standard Hotelling model with quadratic transportation costs (d'Aspremont, Gabszewicz and Thisse, 1979). Hence, media platforms locate at the extremes,  $d_1 = d_2 = 0$ . This shows that the result by Gabszewicz, Laussel, and Sonnac (2001, 2002a) still holds when consumers dislike ads, as long as firms can use unrestricted pay-per-view prices.

**Proposition 1** *In the unique subgame perfect equilibrium of the pay-tv market media platforms maximally differentiate media content. For  $\delta < \alpha^{\max}$ , both platforms provide advertising space according to  $\rho'(a_i) = \delta$  and pass all advertising revenues  $\rho$  onto viewers. For  $\delta > \alpha^{\max}$ , both platforms provide advertising-free programs. Equilibrium profits are neutral in  $\delta$ .*

Consider the uniform case and suppose that  $\delta < 1$ . As mentioned above,  $a_i = (1 - \delta)/2$ . The equilibrium prices are

$$s_i = \frac{1}{4} (4\tau - (1 - \delta^2)).$$

If  $4\tau > 1 - \delta^2$ , that is, there is enough potential content diversity, viewer have to pay a positive price. If the reverse holds, viewers receive a subsidy.

Suppose, on the contrary, that  $\delta \geq 1$ . In this case  $p_i = \tau$ , as in the standard Hotelling model — in fact, this holds outside the uniform case if  $\delta \geq \alpha^{\max}$ .

## 4 Free-to-Air

**Equilibrium for given program content.** Under free-to-air media platform derive revenues only from advertisers.<sup>8</sup> Hence, the profit of media platform  $i$  is  $\pi_i = a_i r_i =$

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<sup>8</sup>Section 4.2 of Anderson and Coate (2003) provide a related analysis for fixed extreme locations, i.e.  $d_1 = d_2 = 0$ .

$N\rho(a_i)b_i$ . The first-order condition of profit maximization then becomes

$$\rho'(a_i)b_i + \rho(a_i)\frac{\partial b_i}{\partial a_i} = 0.$$

Rewriting equation (1) for free-to-air, when  $s_1 = s_2 = 0$ , we have

$$b_1 = \frac{d_1 + 1 - d_2}{2} - \frac{\delta(a_1 - a_2)}{2(1 - d_1 - d_2)\tau}$$

and  $b_2 = 1 - b_1$ . The system of first-order conditions can be written as

$$\rho'(a_i)b_i - \frac{\rho(a_i)\delta}{2(1 - d_1 - d_2)\tau} = 0$$

Note that the second-order condition

$$\rho''(a_i)b_i + 2\rho'(a_i)\frac{\partial b_i}{\partial a_i} < 0$$

is satisfied because  $\rho$  has been assumed to be concave.

At a symmetric equilibrium given symmetry in the content space, i.e.  $d_1 = d_2 \leq 1/2$ ,  $b_i = 1/2$  and advertising levels satisfy

$$\rho'(a_i) = \frac{\rho(a_i)\delta}{(1 - 2d_i)\tau}. \quad (8)$$

This means that for any parameter constellation  $\delta > 0$ ,  $\tau > 0$  and  $d_1 = d_2 < 1/2$  there is a strictly positive amount of advertising that solves (8) because  $\rho(0) = 0$ . Note that the above equation uniquely determines  $a_i$  if  $\rho$  is log-concave and log-concavity is implied by concavity; the concavity of  $\rho$  has been assumed above. It is clear that  $a_i$  cannot be zero in equilibrium so that for  $\delta$  sufficiently large there is overprovision of advertising. To see this, imagine platforms offer zero advertising and do not duplicate content. They make zero profits and market shares are determined according to the positions in the content space. If a platform places a small amount of advertising it will lose some but not all viewers. Hence, it can make positive profits.

In the uniform case, we obtain explicit expressions for the amount of advertising at symmetric locations  $d_1 = d_2$ ,

$$a_i = \frac{1}{2} + (1 - 2d_1)\frac{\tau}{\delta} - \sqrt{\frac{1}{4} + (1 - 2d_1)^2\frac{\tau^2}{\delta^2}} \quad (9)$$

In the special case that  $\delta = 0$ , advertising space is provided according to  $\rho'(a_i) = 0$  as in the monopoly case and the amount of advertising would be same as under pay-tv. In both cases platforms extract full monopoly profits from advertisers as viewers do

not react to them. For any  $\delta > 0$  the amount of advertising is strictly lower than the monopoly. Note that in any equilibrium in which programs are not duplicated, the advertising space is strictly positive because advertising is the only source of revenue. Only if programs are duplicated, i.e.  $d_1 = d_2$ , profits are equal to zero, provided that  $\delta > 0$ . In fact, if platforms share the very same location on the Hotelling line there is an escalation to provide less advertising than the rival to attract viewers. This continues until the market for advertisers is shut down and equilibrium profits are equal to zero. Hence, we should not expect content duplication if  $\delta > 0$ . Different to free-to-air we also would not expect profit neutrality with respect to  $\delta$ .

How does equilibrium advertising depend on the nuisance parameter  $\delta$ ? As stated above, when  $\delta = 0$  advertising would be set at the monopoly level. As  $\delta$  increases, viewers react negatively to ads and we should expect advertising levels to decrease with  $\delta$  as long as platforms do not share the same location. Furthermore, we should expect advertising space to be rather insensitive to  $\delta$  when content is very similar. This is confirmed by the following proposition.

**Proposition 2** *The equilibrium advertising space decreases as advertising becomes more of a nuisance,  $da_i/d\delta < 0$ . However, if programs are of similar content, equilibrium advertising space reacts rather insensitive to the nuisance parameter, i.e.  $\lim_{d_i \rightarrow 1/2} (da_i/d\delta) = 0$ .*

**Proof.** Totally differentiating equation (8) we obtain

$$\begin{aligned}\rho''(a_i)da_i &= \frac{\rho'(a_i)\delta}{(1-2d_i)\tau}da_i + \frac{\rho(a_i)}{(1-2d_i)\tau}d\delta \\ \frac{da_i}{d\delta} &= \frac{\frac{\rho(a_i)}{(1-2d_i)\tau}}{\rho''(a_i) - \frac{\rho'(a_i)\delta}{(1-2d_i)\tau}} = \frac{\rho(a_i)}{(1-2d_i)\tau\rho''(a_i) - \rho'(a_i)\delta}\end{aligned}$$

Since  $\rho$  is concave and, in equilibrium,  $\rho' > 0$ , the amount of advertising necessarily decreases as viewers have a stronger preference against advertising, i.e.,  $da_i/d\delta < 0$ . Taking the limit as  $d_i \rightarrow 1/2$  we have

$$\lim_{d_i \rightarrow 1/2} \left( \frac{da_i}{d\delta} \right) = - \lim_{d_i \rightarrow 1/2} \frac{\rho(a_i)}{\rho'(a_i)\delta},$$

that is, if programs are almost duplicated, the marginal reduction in advertising is  $\rho(a_i)/\delta\rho'(a_i)$ , where the equilibrium advertising space depends on  $d_i$ . From (8) it follows that we can substitute  $\rho(a_i)/\rho'(a_i)$  by  $(1-2d_i)\tau/\delta$ . Hence,

$$\lim_{d_i \rightarrow 1/2} \left( \frac{da_i}{d\delta} \right) = - \lim_{d_i \rightarrow 1/2} \frac{(1-2d_i)\tau}{\delta^2} = 0. \blacksquare \blacksquare$$

In the uniform case, we can also answer how advertising reacts to the nuisance parameter, when this parameter is small, that is, when consumers do not mind much being exposed to advertising. Using the explicit expression for  $a_i$  given by equation (9), we can write

$$\frac{da_i}{d\delta} = -\frac{(1-2d_1)\tau \left[ \sqrt{\delta^2 + 4(1-2d_1)^2\tau^2} - 2(1-2d_1)\tau \right]}{\delta^2 \sqrt{\delta^2 + 4(1-2d_1)^2\tau^2}}$$

Note that for  $\delta$  turning to zero, numerator and denominator both turn to zero. Using the L'Hospital rule, we have

$$\lim_{\delta \rightarrow 0} \frac{da_i}{d\delta} = -\frac{1}{8(1-2d_1)\tau}. \quad (10)$$

This shows that equilibrium advertising reacts strongly to the nuisance parameter  $\delta$  for  $\delta$  small, when platforms offer similar content. We now turn to the equilibrium analysis at the stage where platform choose content.

**The Provision of Program Content.** The relocation tendency is expressed by

$$\begin{aligned} & \frac{\partial \pi_i}{\partial d_i} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} \\ &= N\rho(a_i) \left( \frac{\partial b_i}{\partial d_i} + \frac{\partial b_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} \right) \end{aligned} \quad (11)$$

There is no relocation tendency for interior solutions if the first-order condition at stage 1 holds, which can be written as

$$\frac{\partial b_i}{\partial d_i} + \frac{\partial b_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} = 0 \quad (12)$$

This equation can be expressed as

$$\frac{1}{2} + \frac{\delta(a_i - a_j)}{2(1 - d_i - d_j)^2\tau} + \left( \frac{\delta}{2(1 - d_i - d_j)\tau} \right) \frac{\partial a_j}{\partial d_i} = 0$$

For symmetric locations, this simplifies to

$$\frac{1}{2} + \frac{\delta}{2\tau(1 - 2d_i)} \frac{\partial a_j}{\partial d_i} \Big|_{d_i=d_j} = 0 \quad (13)$$

At symmetric locations, more similar content leads to less advertising in equilibrium. This is stated in the following lemma.

**Lemma 2** *If  $\delta > 0$  then, at a symmetric equilibrium,  $(\partial a_j / \partial d_i) < 0$ .*

**Proof.** To determine the effect of a change in content on advertising space, we have to determine

$$\frac{\partial a_j}{\partial d_i} = - \frac{\frac{\partial^2 \pi_j}{\partial a_j \partial d_i} \frac{\partial^2 \pi_i}{(\partial a_i)^2} - \frac{\partial^2 \pi_i}{\partial a_i \partial d_i} \frac{\partial^2 \pi_j}{\partial a_j \partial a_i}}{\frac{\partial^2 \pi_j}{(\partial a_j)^2} \frac{\partial^2 \pi_i}{(\partial a_i)^2} - \frac{\partial^2 \pi_j}{\partial a_j \partial a_i} \frac{\partial^2 \pi_i}{\partial a_i \partial a_j}}$$

In symmetric equilibrium, i.e.  $d_1 = d_2$ , this can be evaluated as

$$\left. \frac{\partial a_j}{\partial d_i} \right|_{d_i=d_j} = - \frac{2\delta\rho\tau[\delta^2(2-d_1)\rho - \rho''\tau^2(1-d_1)(1-2d_1)^2]}{-\delta^4\rho^2 + (2\delta^2\rho - \rho''\tau^2(1-2d_1)^2)^2} < 0 \quad \blacksquare \blacksquare$$

As a reference point consider the case that viewers do not care about advertising, i.e.  $\delta = 0$ . We find that

$$\frac{\partial \pi_i}{\partial d_i} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} > 0$$

for all  $d_1$ , evaluated at  $d_1 = d_2$ . This means that platforms have a tendency to provide more similar content so that program duplication results. This generalizes the result obtained by Gabszewicz, Laussel and Sonnac (2001, 2002a). However, this result is not robust to introducing advertising as a nuisance. Consider the case  $\delta > 0$ . At  $d_1 = d_2 \rightarrow 1/2$ ,  $\partial a_j / \partial d_i|_{d_i=d_j} = -\tau/\delta$ . Hence, expression (13) simplifies to  $\frac{1}{2} - \frac{1}{2(1-2d_i)}$  that is negative as  $d_i \rightarrow 1/2$ . Therefore, for any  $\delta > 0$  program duplication cannot occur.

**Lemma 3** *If  $\delta > 0$ , program duplication does not occur.*

This shows that the minimum differentiation result is not robust to introducing advertising as a nuisance in the viewers's utility function. The reason goes as follows: if  $\delta > 0$ , the platform with the smaller amount of advertising attracts all viewers under program duplication. Hence, platforms compete in a Bertrand-fashion by reducing the advertising space. In equilibrium, both platforms do not provide advertising space and their revenues are zero. If they differentiate their programs they can make positive revenues.

It is difficult to provide a full characterization of the equilibrium for any parameter constellation in the general case. Still, we can discuss some limiting cases. What happens if the nuisance parameter is large? In this case, maximal differentiation holds, as stated in the following lemma.

**Lemma 4** *If  $\delta$  sufficiently large, media platforms maximally differentiate content.*

**Proof.** In the limit we obtain that

$$\lim_{\delta \rightarrow \infty} \left. \frac{\partial a_j}{\partial d_i} \right|_{d_i=d_j} = - \lim_{\delta \rightarrow \infty} \frac{2(2-d_i)\tau}{3\delta} = 0$$

For large  $\delta$ , the derivative  $\partial a_j / \partial d_i |_{d_i=d_j}$  can be approximated by  $-2(2-d_i)\tau/(3\delta)$ . To obtain an interior solution, the first-order condition at stage 1 has to hold. Expression (13) simplifies to

$$\frac{1}{2} - \frac{2-d_i}{3(1-2d_i)} = 0.$$

However, since the left-hand side is always negative, there is a tendency to offer more differentiated content for symmetric locations. Hence, maximal differentiation occurs — to be more precise, this shows that, for any symmetric equilibrium candidate,  $d_1 = d_2 = 0$ .

This result extends to finite values of  $\delta$  as long as  $\delta$  is high enough. In fact,

$$\begin{aligned} & \left. \frac{\partial \pi_i}{\partial d_i} + \frac{\partial \pi_i}{\partial a_j} \frac{\partial a_j}{\partial d_i} \right|_{d_i=d_j=0} \\ &= \frac{1}{2} - \frac{\delta}{2\tau - \delta^4 \rho^2 + (2\delta^2 \rho - \rho'' \tau^2)^2} \end{aligned}$$

We have to show that the right-hand side of the equation is negative for  $\delta$  sufficiently large. This is equivalent to

$$\frac{1}{2} < \frac{\delta^2 \rho [2\delta^2 \rho - \rho'' \tau^2]}{-\delta^4 \rho^2 + (2\delta^2 \rho - \rho'' \tau^2)^2}$$

For large  $\delta$  the right-hand side is approximately  $2/3$ . Equivalently, the previous inequality holds if  $\delta > \tau \sqrt{(1 + \sqrt{2})(-\rho'')/\rho'}$ , which is a finite value since at equilibrium  $\rho'$  is positive. ■

For lower values of the disutility parameter we can show that the location of content is monotonically decreasing in  $\delta$ . From expression (13) we conduct comparative statics obtaining:

$$\begin{aligned} & \text{sign} \left( \left. \frac{\partial d_i}{\partial \delta} \right|_{d_i=d_j} \right) \\ &= \text{sign} \left( 2\rho''(1-2d_i)^3 \tau^2 [\delta^4 \rho^2 (5-d_i) \right. \\ & \quad \left. - 2(1-2d_i)^2 (2-d_i) \delta^2 \tau^2 \rho \rho'' + (1-2d_i)^4 (1-d_i) \tau^4 \rho''^2] \right) < 0. \end{aligned}$$

We summarize our findings in the following proposition.

**Proposition 3** *In subgame perfect equilibrium of the free-to-air market, media platforms do not minimally differentiate media content for  $\delta > 0$ . Content differentiation is increasing in  $\delta$  and reaches maximal differentiation for  $\delta$  sufficiently large.*

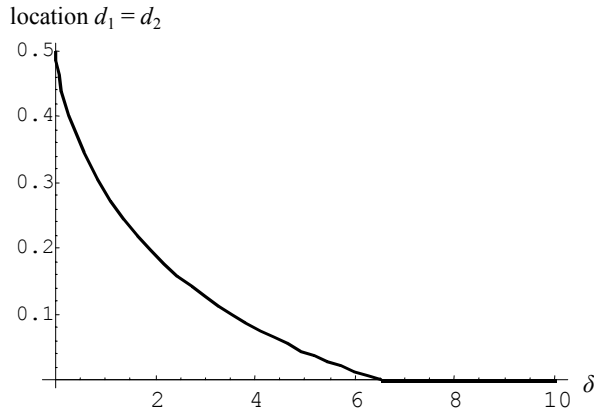


Figure 1: Content provision under free-to-air

We can also ask what happens as viewers view programs of different content as hardly substitutable, i.e., as  $\tau$  becomes large. In the limit we obtain

$$\lim_{\tau \rightarrow \infty} \left. \frac{\partial a_j}{\partial d_i} \right|_{d_i=d_j} = \lim_{\tau \rightarrow \infty} \frac{2(1-d_i)\delta\rho}{(1-2d_i)\tau\rho''}$$

For large  $\tau$  the sign of the relocation tendency has the same sign as

$$\frac{1}{2} - \frac{2(1-d_i)\delta^2\rho}{(1-2d_i)^3\tau^2\rho''}$$

For any finite  $\delta$ , the parameter  $\tau$  can be chosen sufficiently large such that for any content provision  $d_i$  maximal differentiation never arises and platforms have a tendency to provide more similar content.

Notice that equilibrium profits decline with  $\delta$ . This is because profits arise only from advertising and in a symmetric equilibrium each platform has 50% of the viewers. Thus profits are  $N\rho(a_i)/2$ .  $\rho(a_i)$  is maximized when  $\rho'(a_i) = 0$  which from (8) occurs only when  $\delta = 0$ . Recalling that  $\partial a_i/\partial\delta < 0$ , it follows immediately that equilibrium profits are monotonically decreasing in  $\delta$ ,  $\partial\pi_i/\partial\delta = N(\rho'(a_i)/2)(\partial a_i/\partial\delta) < 0$ .

To explicitly determine equilibrium content, we return to the uniform case. In line with the general case just described, for  $\delta = 0$  there is program duplication and for  $\delta$  sufficiently large there is maximal content differentiation. Figure 1 gives the equilibrium programming content for  $\tau = 1$ : for  $\delta > 6.53$  there is maximal differentiation,<sup>9</sup> for lower  $\delta$  the differentiation between programs,  $1 - 2d_i$ , is a concave function in  $\delta$ .

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<sup>9</sup>To be precise, the critical  $\delta = 2\sqrt{5 + 4\sqrt{2}}$ .

## 5 Pay-Tv Markets versus Free-to-Air-Markets

### 5.1 Advertising and Content Comparison

We start by comparing pay-tv and free-to-air for exogenous content provision.

**Exogenous content provision.** In pay-tv advertising is given by (7) while in free-to-air market it is given by (8). Since a monopolist would set  $\rho'(a_i) = 0$ , it follows that competing media platforms provide less advertising than a monopolist. Only at  $\delta = 0$ , the amount of advertising space is the same under pay-tv and free-to-air and coincides with the advertising provision of a monopolist. When  $\delta > \alpha^{\max}$ , then pay-per-view implements the first-best advertising allocation, which in this case is 0 advertising, whereas free-to-air leads to a strictly positive amount of advertising, provided that content is not perfectly duplicated. In other words, if viewers strongly dislike ads, then pay-tv has necessarily less advertising. For lower values of  $\delta$ , if  $\rho(a_i) > (1 - 2d_i)\tau$  then free-to-air has again more advertising than pay-tv. The general comparison between the two systems also depends on content: while content does not matter for advertising space in pay-tv, it does affect advertising in free-to-air. Under perfect content duplication, advertising is zero for any value of  $\delta$  in free-to-air: in this case pay-tv either provides the same zero amount when  $\delta > \alpha^{\max}$ , or it strictly provides more advertising.

In the uniform case we make a more detailed comparison of equilibrium advertising depending on  $\delta$ ,  $\tau$ , and  $d_1 (= d_2)$ . Advertising with free-to-air is always greater than advertising with pay-tv if  $\delta > 1$ . If  $\delta < 1$  it is also greater if the following inequality holds:

$$\frac{1}{2} + (1 - 2d_1)\frac{\tau}{\delta} - \sqrt{\frac{1}{4} + (1 - 2d_1)^2\frac{\tau^2}{\delta^2}} > \frac{1 - \delta}{2}$$

which can be re-written as:

$$d_1 < \frac{1}{2} - \frac{1 - \delta^2}{8\tau}$$

This inequality is likely to hold if programs are not easily substitutable ( $d_1$  small and  $\tau$  large). Conversely, if programs are substitutable, competition for viewers under free-to-air makes platforms choose a restrictive advertising policy. This leads to less advertising than under pay-tv.

**Endogenous content provision.** Pay-tv always ends up providing maximal content differentiation, while free-to-air provides less diversity of content. Only when  $\delta$  is sufficiently high both systems provide the same (maximal) content diversity. Pay-tv content diversity is always excessive in terms of social welfare. On the other hand, there may be socially too little content diversity (if viewers do not strongly dislike ad) or excessive diversity (if viewers strongly dislike ads) under free-to-air. In the previous

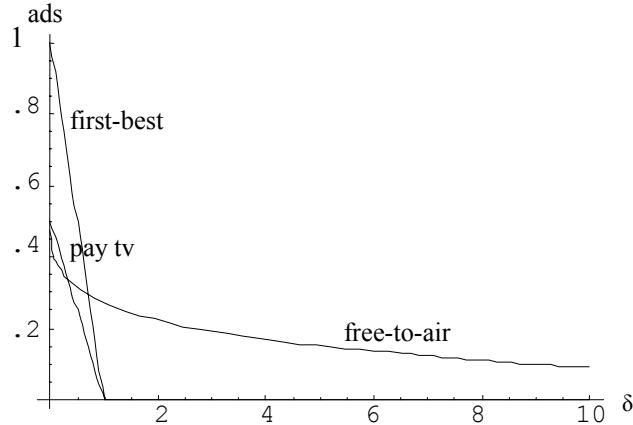


Figure 2: Advertising intensity with endogenous content

section we showed that location under free-to-air is monotonically decreasing between 0.5 and 0 as  $\delta$  increases. hence there is always one particular value of  $\delta$  such that content provision under free-to-air is socially optimal ( $d_i = 1/4$ ).

Advertising comparison is a special case of the previous analysis once the location of content is endogenized. We illustrate the provision of advertising space in the uniform case (see Figure 2 for  $\tau = 1$ ).

For  $\delta = 0$ , both systems offer the same (monopoly) advertising level. As  $\delta$  is slightly increases, the chosen programs are close substitutes under free-to-air whereas there is maximal program differentiation under pay-tv. The slope of the function  $a_i(\delta)$  is  $-1/2$  under pay-tv whereas it is  $-\infty$  under free-to-air, when evaluated at  $\delta = 0$ . This extreme sensitivity of advertising arises from the endogenous location of free-to-air: at  $\delta = 0$  both platforms perfectly duplicate content, but as  $\delta$  increases they differentiate a bit, otherwise their advertising level would drop immediately from the monopoly level (when  $\delta = 0$ ) to zero (when  $\delta > 0$ ). When  $\delta$  is very small, programs are “almost” duplicated and advertising reacts very sharply to  $\delta$ . Hence, for the nuisance parameter  $\delta$  sufficiently small, there is less advertising under free-to-air than under pay-tv. Only when viewers strongly dislike advertising does pay-tv lead to less advertising. Since there is always underprovision of advertising under pay-tv in the case that the socially optimal amount of advertising is strictly positive, there is an intermediate range of values for  $\delta$  such that the amount of advertising under free-to-air is socially better than under pay-tv. We summarize our findings with respect to equilibrium advertising with endogenous content provision in the following proposition.

**Proposition 4** *In the uniform case, advertising with endogenous content provision is less under free-to-air than under pay-tv if  $\delta$  is sufficiently small, and greater if  $\delta$  is sufficiently large. In the former case, social underprovision of advertising is more*

*pronounced under free-to-air; in the latter case, free-to-air can lead to under or over-provision of advertising.*

## 5.2 Welfare Comparison

**Welfare with exogenous content provision.** For given content, the only difference in welfare under the two pricing schemes comes from different amounts of advertising. Our earlier results in section 5.1 with respect to advertising levels then directly translate into welfare results. In particular, for large  $\delta$  welfare is higher under pay-tv than under free-to-air because there is overprovision of advertising under free-to-air whereas the advertising level is socially optimal under pay-tv. For  $\delta$  sufficiently low, there is under provision of advertising under the two pricing schemes. This underprovision is more pronounced under free-to-air when exogenous content of the two platforms is quite similar: in this case pay-tv is again socially desirable. However, there is an intermediate range of values for  $\delta$  and enough content differentiation such that there is severe underprovision of advertising under pay-tv whereas the advertising level is close to first-best levels under free-to-air. On this range, free-to-air is socially desirable.

In the uniform case, welfare under a pay-tv system and free-to-air are respectively:

$$\begin{aligned} W_{pay-tv}^{ad} &= \frac{3N}{8}(1 - \delta)^2 \text{ if } \delta < 1 \\ W_{free-to-air}^{ad} &= N(a_i(1 - \delta) - a_i^2/2) \text{ where } a_i \text{ is given by (9)} \end{aligned}$$

Figure 3 plots the results of the welfare comparison for  $\tau = 1$ . In line with the general case, three regions arise. Free-to-air is preferred to pay-tv only if the nuisance parameter is not too high and there is sufficient content diversity.

**Welfare with endogenous content provision.** The welfare comparison is somewhat more involved if platforms choose content. In this case also the welfare effect of content has to be taken into account. However, we can make a number of observations. For  $\delta$  large ( $\delta > \max\{\alpha^{\max}, \tau\sqrt{(1 + \sqrt{2})\rho''/\rho'}\}$ ), platform maximally differentiate content under both pricing schemes. In both cases there is the same (excessive) content diversity. Hence, only advertising matters for the welfare comparison: pay-tv is socially desirable for  $\delta$  large since it provides zero advertising while free-to-air overprovides it. For  $\delta$  sufficiently low, free-to-air platforms do not maximally differentiate content so that free-to-air is socially preferred to pay-tv as far as content is concerned. Since on an intermediate range of  $\delta$  it is socially preferred also with respect to advertising, free-to-air leads to higher welfare than pay-tv for intermediate values of  $\delta$ . The main question is which of the two schemes leads to higher welfare for  $\delta$  very small. Clearly, at  $\delta = 0$  both lead to the same welfare because in our model program duplication and maximal program differentiation involve the same welfare loss compared to the first-best and advertising levels under both schemes are equal to the monopoly advertising

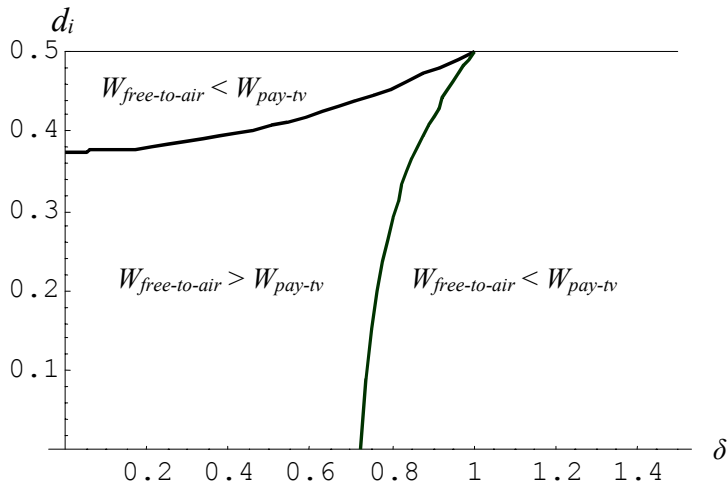


Figure 3: Welfare comparison for given differentiation of content

level. For very small  $\delta$ , the advertising level under pay-tv is socially preferred but content provision under free-to-air is socially preferred. It has been argued above that, for given content, there is a more pronounced underprovision of advertising under free-to-air when content is almost perfectly duplicated. This still holds true under endogenous content provision: as  $\delta$  is increased slightly above 0, a free-to-air platform changes its content by a very small amount while it decreases sharply the number of ads it shows. Hence the welfare gain from better content is limited, while the under-provision of ads is exacerbated: pay-tv dominates over free-to-air for small nuisance is transportation costs are not too high. This is illustrated for the uniform case in Figure 4 ( $\tau = 1$ ) In the case of the figure, pay-tv has better welfare properties for  $\delta < 0.127$  and for  $\delta > 1.101$ .

If transportation costs are small, then pay-tv is preferred for a wide range of nuisance parameters. As can be seen from figure 5, the line in the  $\delta$ - $\tau$  space such that welfare under free-to-air is the same as welfare under pay-tv is U-shaped.

If transportation costs are large, the nuisance parameter must be very small or very large for pay-tv to give higher welfare than free-to-air. For instance, at  $\tau = 100$ . For instance, if  $\tau = 100$ , then free-to-air is preferred for any  $\delta$  with  $0.000084 < \delta < 4.149$ . We summarize our main findings in the following proposition.

**Proposition 5** *In the uniform case, welfare with endogenous content provision is greater under pay-tv, if, for given  $\tau$ , the nuisance parameter  $\delta$  is sufficiently small or sufficiently large and if, for given  $\delta$ , the potential differentiation between programs  $\tau$  is sufficiently small.*

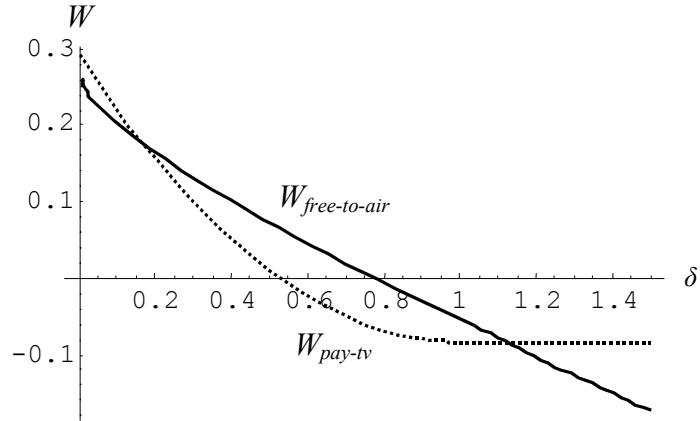


Figure 4: Welfare under pay-tv versus free-to-air for endogenous content

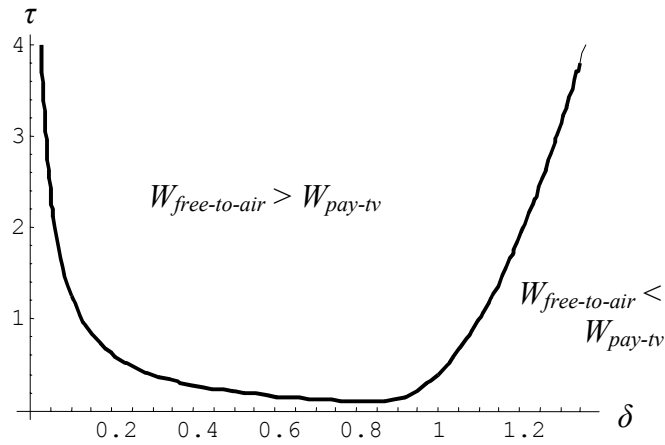


Figure 5: Welfare comparison in the  $\delta$ - $\tau$  space

## 6 Conclusions and discussion

[to be written]

**The Role of Expectations.** Some works have used the concept of "fulfilled" expectations, instead of solving the system of equations (1), (2) in the last stage of the game. This alternative approach would simplify calculations but also lead to very peculiar results where the nuisance parameter does not play any role. Fulfilled expectations, in fact, cut any direct link between the two sides of the market. Hence the advertising space would always be set at the pure monopoly level (1/2 in the uniform example), independently of  $\delta$ , both under free-to-air and under pay-tv. In the first stage, the location game would also be very simple: maximal differentiation would arise in pay-tv and minimal differentiation under free-to-air. As a consequence, total welfare would always be identical under both systems! As one departs from fulfilled expectations, all these results would not be robust.

**Advertising Space and Advertising Prices.** A similar observations applies when the advertising rate is set exogenously, as assumed, for instance, by Gabszewicz, Laussel, and Sonnac (2004). Once again, there is no direct link between the two sides of the market. When advertising rates are set by platforms instead, as we have already mentioned, our results would go unaltered if the choice variable is the advertising rate per viewer. On the other hand, the analysis would get more complicated if platforms set lump sum advertising rates. This is left for further research.

**Advertising as a Complement.** tbw

**Advertising Ban or Restrictions.** In our model, bans in a pay-tv would make no sense as there is always underprovision of advertising (but content regulation could make sense in this environment). Bans or restrictions on advertising make sense in free-to-air if  $\delta$  is high.

**Markets for Media Content.** tbw

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